High-rate, Full-Diversity STBCs from Field Extensions

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Abstract — **We present a construction of high-rate codes from field extensions that are full-rank also. Also, we discuss the coding gain and decoding of these codes.**

I. High-rate full-diversity codes

Let $\mathbb{Q}$ denote the field of rational numbers and let S be the signal set, over which we want rate R (complex symbols/channel use) Space-Time Block Codes (STBCs) for n transmit antennas. An $n \times n$ STBC is said to be over S if all the codeword matrices have entries that are complex linear combinations of elements of S. Consider the following chain of field extensions:

$$\mathbb{Q} \subset \underbrace{\mathbb{Q}(S)}_{L} \subset \underbrace{\mathbb{Q}(S,\alpha)}_{F} \subset \underbrace{\mathbb{Q}(S,\alpha,\beta)}_{K}$$

with $[F:L] = R$ and $[K:F] = n$. Then we have Theorem 1 the proof of which is an extension of the proofs given in [1, 2] for similar theorems:

Theorem 1 *The set of matrices of the form* $f_0(\alpha)I + f_1(\alpha)M_\beta + f_2(\alpha)M_\beta^2 + \ldots + f_{n-1}(\alpha)M_\beta^{n-1}$ *where* M_β *is the companion matrix of* β *over* F *and* $f_i(\alpha)$ *is a* $R-1$*-th degree polynomial in* α *with coefficients from* L*, has the property that the difference of any two distinct matrices in the set is of full rank and hence, constitutes a rate-*R*, full-rank STBC over* S *if the coefficients of* $f_i(\alpha)$ *are restricted to* $S \subset L$.

In particular, if the minimal polynomial of $\beta \in K$ over F is of the form $x^n - \gamma$, then the codeword matrices are of the form

$$\begin{bmatrix} f_0(\alpha) & \gamma f_{n-1}(\alpha) & \cdots & \gamma f_1(\alpha) \\ f_1(\alpha) & f_0(\alpha) & \cdots & \gamma f_2(\alpha) \\ \vdots & \vdots & \ddots & \vdots \\ f_{n-1}(\alpha) & f_{n-2}(\alpha) & \cdots & f_0(\alpha) \end{bmatrix}$$

where $f_i(\alpha) = \sum_{j=0}^{R-1} f_{i,j}\alpha^j$ with $f_{i,j} \in S$. In the above development, the selection of α and β depends on the signal set, R and n, which restricts the coding gain of the STBC. By choosing α to be a transcendental element over $\mathbb{Q}(S)$, the polynomials $f_i(\alpha)$, can be of arbitrary degree and hence we get a rate-R, full-rank STBC for any arbitrary $R \geq 1$.

II. Coding gain

Given any field extension K/F, let $N_{K/F}(k)$ denote the algebraic norm from K to F of $k \in K$. Then, we have the following theorem:

Theorem 2 *Let* $K = F(\beta)$ *be an* n*-th degree extension of* F*. The coding gain of the STBC over* S *constructed as in Theorem 1 is given by* $\min_{k \neq k'} |N_{K/F}(k-k')|^{2/n}$*, where* $k = f_0 + f_1\beta + \cdots + f_{n-1}\beta^{n-1}$ *and* $k' = f'_0 + f'_1\beta + \cdots + f'_{n-1}\beta^{n-1}$.

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Theorem 3 *Let* S *be a signal set carved out from* $\mathbb{Z}[j]$*. Let* K *and* F *be as in Theorem 2, with* β *integral over* F*, i.e., minimal polynomial of* β *over* F *is in* $\mathbb{Z}[j][x]$*. Then the coding gain of STBCs constructed as in Theorem 1 with* $R = 1$*, is*
(i) 1 if S *is contains two nearest neighbors of* $\mathbb{Z}[j]$
(ii) $4d^2$ *if* S *is a possibly rotated regular QAM signal set* $\{((2k-1-Q)d + j(2l-1-Q')d)e^{j\phi} | k \in [1,Q], l \in [1,Q']\}$ *for any* $\phi \in [0, 2\pi]$.

III. Decoding and Simulations

The STBC's constructed in Theorem 1 are sphere decodable when S is a lattice constellation or a PSK signal set. We present simulation results for several such STBCs over QAM-signal sets with $R = 1$ obtained with the monic irreducible polynomials, $x^3-1-j, x^4-j, x^5-1-j, x^6-1-j, x^7-1-j, x^8-j$ respectively for 3,4,5,6,7 and 8 transmit antennas in Figure 1. The number of receive antennas for 3 and 4 antennas is 2 and for the rest it is 1. We use sphere decoding to decode our STBCs. The curves match with those given in [3] in all the cases which means these codes are of maximum coding gain.

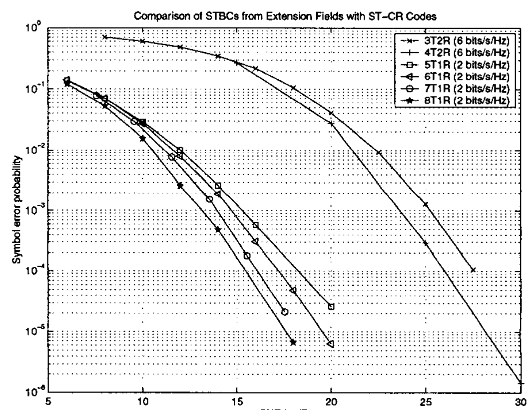

Figure 1: Comparison of STBCs from field extensions with ST-CR Codes of [3]

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