Dirichlet-to-Neumann Map for Evolution PDEs on the Half-Line with Time-Periodic Boundary Conditions

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This talk is about a new approach to the asymptotic analysis for large times of the generalised "Dirichlet-to-Neumann map", i.e., the problem of determining the unknown boundary values in terms of the given initial and boundary conditions, for evolution PDEs on the half-line with time-periodic boundary conditions [1]. Our analysis includes linear and integrable nonlinear PDEs; these types of equations share the property that they can be written as the compatibility condition of a set of linear equations, the so-called Lax pair [2,3]. We show that the time-dependent part of the Lax pair yields the large t asymptotics of the periodic unknown boundary values in terms of the given periodic boundary data via an elegant and simple algebraic calculation. Using the time-dependent part of the Lax pair we construct the "Q-equation" on which the explicit determination of the unknown boundary values is based. The Q-equation was first introduced by Lenells and Fokas for the Nonlinear Schrödinger (NLS) equation [4], but we simplify the techniques of this paper and we show the usefulness of the Q-equation approach also for linear equations. The latter is a very interesting development, since it illustrates the power of Lax pairs even in the case of linear problems. Lax pairs were a crucially important ingredient of the Fokas Method [5] and it is noteworthy that in this problem their importance is re-confirmed. Our analysis of the generalised Dirichlet-to-Neumann map via an appropriately constructed Q-equation will focus mainly on linear equations including the heat equation, the diffusion-convection equation, the linearised KdV equation and the linearised version of the NLS equation. We also show that the Q-equation provides a straightforward way for deriving the remarkable results of Boutet de Monvel, Kotlyarov and Shepelsky for the focusing NLS equation [6].

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