A Realistic Small-World Model for Wireless Mesh Networks

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***Abstract*—Small-world network concept deals with the addition of a few Long-ranged Links (LLs) to significantly bring down the average path length (APL) of the network. The existing small-world models do not consider the real constraints of wireless networks such as the transmission range of LLs, limited radios per mesh router, and limited bandwidth for wireless links, therefore, we propose C-SWAWN (Constrained Small-World Architecture for Wireless Network) model for Wireless Mesh Networks (WMNs). We then propose three LL addition strategies for reducing APL to the centrally placed Gateway node in WMNs. In moderately large WMNs, a 43% reduction in APL to Gateway can be achieved with the addition of 10% LLs (with respect to number of mesh routers) in our C-SWAWN model with greedy LL addition strategy. Detailed studies show realistic performance benefits with application of small-world concept in WMNs.**

***Index Terms*—Long-ranged links, wireless mesh networks, average path length, network architecture, small-world networks.**

I. INTRODUCTION

A WIRELESS mesh network (WMN) consists of stationary mesh routers that use multi-hop wireless relaying for providing communication services to wireless clients. There are several advantages for WMNs: self configurability, high fault-tolerance, and high network deployment flexibility. However, WMNs suffer from many disadvantages. Rapid throughput degradation with path length, poor capacity scaling with larger networks, and wireless channel related performance issues are some examples. One method that can help reduce the path length, thereby provide better capacity and end-to-end delay, is the concept of small-world networks [1].

A. Network Models and Related Work

A small-world network is characterized by a high Average Clustering Coefficient (ACC) similar to that of regular networks and a low Average Path Length (APL), similar to that of random networks. In Watts-Strogatz (WS) model [1], a small-world network is constructed by probabilistically *rewiring* the network links. In Newmann-Watts (NW) model [2], new Long-ranged Links (LLs) are added with probability p. In Kleinberg model [3], the probability of having an LL between two nodes is inversely proportional to their euclidian distance. In [4]–[6], the authors studied the application of small-world concept in wireless networks. However, the existing small-world models [1]–[6] have many shortcomings for application in WMNs. The WS model is impractical for WMNs because it requires rewiring existing links, which is very complex to achieve in wireless networks. While one can apply NW and Kleinberg models in the wireless context, they do not consider the limit of number of radios or their bandwidth, that constrains the number of LLs and the traffic they can carry. Further, wireless networks cannot realize LLs of arbitrary lengths due to technological, transmit power, or cost constraints.

The authors of [4] and [5] studied small-world network benefits in wireless networks by adding LLs between randomly chosen node pairs whose distance is from $[2, r]$ hops, where r is the maximum distance in hops. However, such an LL addition method is not easy to realize in WMNs because for wireless links, the farthest point they would reach would be determined by the transmission range of radio interfaces. Since there is a bound on maximum length of LL, it may not possible to establish LLs that exceed this bound, even if a node pair is well within the allowed r hops. Therefore, in our work we use euclidean distance, an approximation of transmission range, which is a better criteria for limiting the length of LLs. In [6], a few *short cut wires* are added to improve energy efficiency of wireless sensor networks. Their model is quite different from the above discussed models (and our model) because they use wires for realizing LLs and assume each LL originates from the sink node. All these existing works did not thoroughly study the benefits of small-world network concept in WMNs by considering the constraints in a WMN on the number of LLs per network, transmission range of LLs, bandwidth of the LLs, the number of LLs a node can have, and the distance from gateway nodes. Hence, we propose a new model, Constrained Small-World Architecture for Wireless Network (C-SWAWN), for WMNs and then present realistic performance results.

II. OUR SMALL-WORLD NETWORK MODEL FOR WMNS

Let N denote number of WMN routers (or nodes) placed uniformly at random locations in a given two-dimensional terrain area. We consider one Gateway node which is placed in the weighted center of WMN. We assume that WMN routers and Gateway node are stationary and therefore do not face any strict power constraints. Each node except Gateway consists of two kinds of radios: one short-range radio for communication with its one-hop neighbors and K_{LL} long-range radios for establishing point-to-point LLs with far away nodes. The proposed model can be represented as C-SWAWN(R_S, R_L, K_{LL}). Here R_S denotes the transmission range of short-range radios. The R_L denotes the maximum range of long-range radios. The parameter K_{LL} denotes the number of long-range radios per node that limits the number of LLs a node can typically establish. We assume that the Gateway node does not have any LL radios and, therefore, all traffic has to reach the Gateway through its one-hop neighbors.

Manuscript received February 12, 2010. The associate editor coordinating the review of this letter and approving it for publication was S. Gupta.

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Digital Object Identifier 10.1109/LCOMM.2011.020111.100266

We also assume that an LL can be realized between a pair of nodes with help of highly directional point-to-point radios and, therefore, any two LLs do not interfere with each other. Also LLs are bi-directional and their assignments do not change dynamically. In this model, we assume shortest path routing (in terms of hops) and traffic from all wireless clients is routed through mesh routers in a multi-hop fashion to the Gateway node. APL from mesh routers to the Gateway (G-APL, measured in terms of hops) is hence an appropriate metric for analyzing the benefits of small-world concept in WMNs. Actual APL from wireless clients to the Gateway is (G-APL+1) hops. We also measure Average Clustering Coefficient (ACC), Average Neighbor Degree (AND), and Call Acceptance Ratio (CAR). The neighbor degree of a node gives number of nodes present in its transmission range. The clustering coefficient of a node tells what fraction of its friends (one-hop neighbors) are friends of each other as well. It is the ratio between the number of edges that actually exist among its neighbors and the total possible number of edges that can exist among them. The CAR is defined as the ratio between the number of calls admitted into the network and the number of calls attempted by nodes.

A. *LL addition Strategies*

We propose three LL addition strategies. According to the basic strategy, called **Random LL addition strategy (RAS)**, a pair of nodes (i and j) is randomly chosen and checked for certain constraints for adding an LL between them. The first constraint is that the euclidean distance r between nodes i and j lies between R_S and R_L and the second constraint is that both nodes must have at least one LL radio un-assigned to any other LL. In our second strategy, called **Gateway aware LL addition strategy (GAS)**, to ensure that each added LL results in improved G-APL, besides the two constraints defined in RAS, we impose an additional constraint: $|d(i)-d(j)| \geq \Delta_h$, where $d(i)$ is the number of hops between node i and the Gateway through the shortest path and Δ_h is the minimum difference in the shortest paths to Gateway for any two nodes we want to connect by an LL. Δ_h is a controllable parameter whose minimum value is two. The process of adding LLs is repeated till either we reach the limit on the number of LLs attempted for addition in WMN (N_{LL}) or the network saturates with LLs, (*i.e.,* we cannot add any more LLs due to constraints on R_L, K_{LL}, and Δ_h). The number of LLs beyond which we cannot add any more LLs in a WMN is called network saturation point (N_{sat}).

Finally, as a modified form of our second scheme, we propose **Gateway aware greedy LL addition strategy (GAGS)** where LLs corresponding to the highest Δ_h value are added first and if the network reaches N_{sat}, Δ_h is relaxed (reduced by one till it reaches 2) to add more LLs and the process goes on to add the desired number of LLs (N_{LL}) in the network.

An example of small-world WMN given by GAGS scheme is given in Figure 1. We have used $N = 50, N_{LL} = 6, K_{LL} = 2,\ R_S = 180m, and\ R_L = 600m$. The G-APL reduced from 2.3 hops to 2 hops after adding 6 LLs.

B. *Slot Allocation and Bandwidth Reservation*

In order to study how small-world network concept helps in improving WMN capacity, we implemented a call admission control scheme on top of our C-SWAWN model. Every node has B1 time-slots available for its short-ranged radio. Similar to the TDMA mechanism in [7], for a node j to transmit in a particular slot through a short-ranged link, the slot must be free at node j and none of the nodes lying in its transmission range must be receiving in that slot from their neighbors. An LL, however, is a point-to-point connection with directional antennas and hence it is assumed that no two LLs interfere with each other. Every node has B2 time-slots for each LL. Earliest feasible allocations are made for each hop in increasing order of free slots available for the hop (bottleneck hops first). Calls are attempted to be established between random node pairs and for any call, if any hop on shortest path does not have free slots, the call is dropped and the slots allotted, so far, for that call are released.

III. PERFORMANCE RESULTS

We study performance of the LL addition strategies on C-SWAWN model, in terms of metrics G-APL, APL, ACC, AND, and CAR, by carrying out detailed experiments, in a simulator platform developed using MATLAB. We take a WMN with 400 nodes distributed in uniformly random fashion over an area of 2000 m × 2000 m. The results shown in plots are averaged over 20 seeds and drawn with 95% confidence intervals. Unless otherwise mentioned, the values used for simulations are $K_{LL} = 2$, $R_S = 200\ m$, and $R_L = 800\ m$. Figure 2 shows variation of G-APL with N_{LL} (number of LLs attempted for addition) for all LL addition schemes. G-APL decreases with increase in number of LLs added for all the schemes. GAS scheme with higher Δ_h outperforms GAS schemes with lower Δ_h for a given number of added LLs as higher Δ_h has more potential to bring down G-APL. The GAGS scheme performs better than all other schemes because it adds LLs for all possible Δ_h values by start greedily adding first with the highest Δ_h. For an addition of 40 LLs, we observe improvement in G-APL for RAS, GAS ($\Delta_h = 2$), GAS ($\Delta_h = 4$), and GAGS schemes as 25.4%, 27%, 37.9% and 43.3%, respectively, compared to the unmodified network. We present rest of the performance results only using GAGS scheme as it out performs all other schemes.

We also conducted experiments to study small-world benefits in 20 × 20 grid topology. G-APL drops by 42% after addition of 40 LLs in grid topology. Initial G-APL (*i.e.,* when no LLs are added) is slightly higher for random topology (5.415) compared to grid topology (5.237), but the final G-APL of both comes to be nearly the same (3.07).

Figure 3 shows the variation of G-APL, APL, ACC, and AND with N_{LL}. An addition of 40 LLs bring down the G-APL by 43% without significantly affecting ACC (ACC drops by less than 4%). Note that for a WMN with 400 nodes, 40 LLs might appear only 0.05% of total possible links but this is actually 10% with respect to the total number of nodes and is, therefore, not a small number. *We particularly note, in this context, that in [4], it was mentioned that a very small percentage (0.2% with respect to total possible links)*

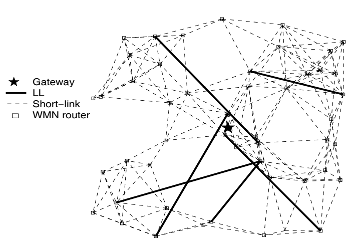

Fig. 1: A small-world WMN.

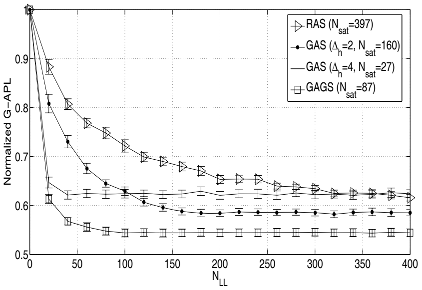

Fig. 2: G-APL vs N_{LL}.

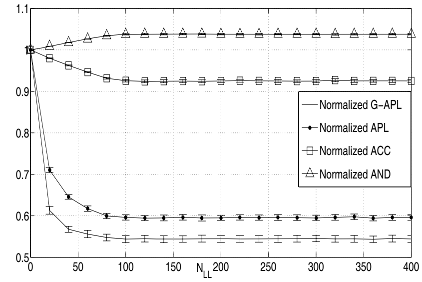

Fig. 3: G-APL, ACC, AND vs N_{LL}.

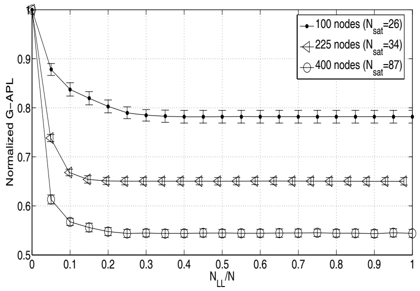

Fig. 4: G-APL vs ratio of LLs to N.

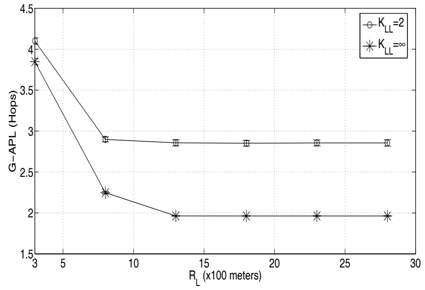

Fig. 5: G-APL vs R_L vs K_{LL}.

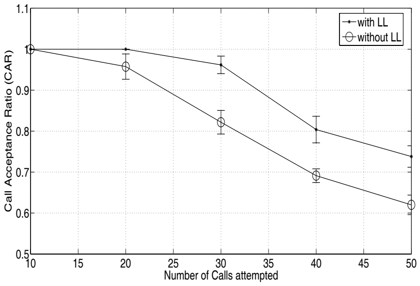

Fig. 6: CAR vs number of calls.

of additional LLs results in significant decrease (25%) in APL for wireless networks. This 0.2% refers to a large number of LLs with respect to the total number of nodes.

We observe a higher improvement in G-APL for larger networks (Figure 4). The main reason is that the G-APL of an unmodified WMN is higher for a larger network and this leaves a larger scope of improvement in G-APL for such networks. Figure 5 shows variation of G-APL with R_L when 420 LLs are attempted to be added. A higher R_L relaxes the constraint leading to longer LLs on an average. This effect is reflected in the improvement in G-APL as R_L increases. However, G-APL saturates beyond certain R_L for both cases of K_{LL} (2 and ∞). These results suggest that R_L should be at most 4 to 6 times R_S. For $K_{LL} = 2$, N_{sat} is 90 LLs where as $K_{LL} = \infty$ case has N_{sat} of 362 LLs. For the latter, most of the nodes in the network are either a neighbor of Gateway or get a direct LL to a neighbor of Gateway (Gateway does not have any LL radios) and hence G-APL is around 2 hops.

In Figure 6, we plot CAR vs the number of attempted calls for a WMN without LLs and a WMN with 10 LLs. The CAR drops as more calls are attempted in the network due to the limited link bandwidth. The network with LLs has better CAR than the network without any LLs. The performance gain remains even when we increased the bandwidth for short-ranged radios in case of the network without any LLs to $B1^{'} = B1 + \frac{10\times B2}{N}$ as a way to ensure fairness in the comparison by keeping total bandwidth of the networks constant in both WMNs. *Experiments also reveal that LL radios do not require as much bandwidth as short-ranged radios and only a fraction (15% to 20%) of B1 is sufficient for B2.* This is because short-ranged links connected to a node, facing multi-hop interference, saturate faster than LLs.

IV. CONCLUSIONS

We proposed C-SWAWN model, a small-world WMN model that considers real-world constraints. Further we proposed three LL addition strategies for C-SWAWN model. We found that significant performance benefits, in terms of G-APL and CAR, can be achieved in WMNs by applying small-world network concept even with real constraints of wireless networks. Contrary to the expectations, we found that the LLs do not form bandwidth bottlenecks.

ACKNOWLEDGEMENTS

This work was supported by the UCSD CWC and the WIISARD-SAGE project sponsored by National Library of Medicine and National Institutes of Health.

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