Vector Calculus

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Length of plane curves

Area of surfaces of revolution

Length of the basic element

$$= \sqrt{((x+h)-x)^2 + (f(x+h)-f(x))^2}$$

$$= h\sqrt{1 + \left(\frac{f(x+h)-f(x)}{h}\right)^2}$$

1. Lenth of a smooth (continous and differentiable) function $y = f(x)$ from $x = a$ to $x = b$ is

$$\begin{aligned} L &= \int_a^b \sqrt{1 + f'(x)^2} dx \\ &= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\ &= \int_a^b ds \end{aligned}$$

$ds = \sqrt{dx^2 + dy^2}$ is called the arc length differential

Similar expression for $x = g(y)$

$$\begin{aligned} L &= \int_c^d \sqrt{1 + g'(y)^2}\, dy \\ &= \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \\ &= \int_c^d ds \end{aligned}$$

Area of surface revolving a smooth curve $y = f(x)$ about the x-axis

$$S = \int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2}dx$$

$$= \int_a^b 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx$$

- Area of surface revolving a smooth curve $x = f(y)$ about the y-axis

$$S = \int_c^d 2\pi f(y)\sqrt{1 + f'(y)^2}dy$$
$$= \int_c^d 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}dy$$

- In short form, with ρ as radius function,

$$\int_a^b 2\pi \rho ds$$

- If f is not positive then consider $|f(x)|$