Vector Calculus

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January 6, 2016

Volume by slicing

The volume of the solid with integrable cross section area $A(x)$ between $x = a$ and $x = b$ is given by

$$\text{Volume} = \int_a^b A(x)dx$$

Cavalieri's Principle

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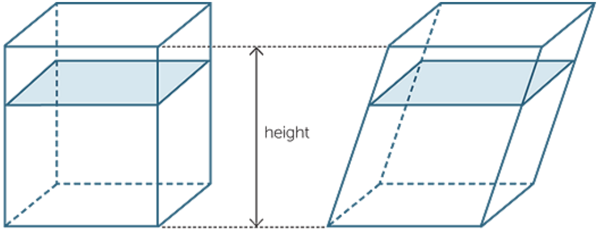
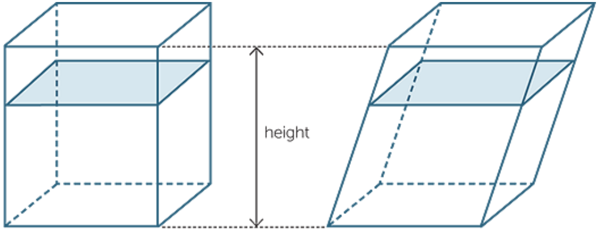


Cavalieri's theorem (1647). Solids with same altitude and same cross section areas at same level have same volume.

Cavalieri's Principle



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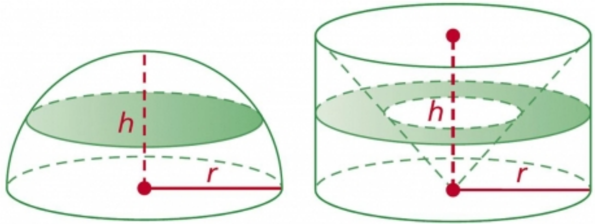
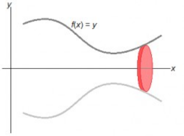
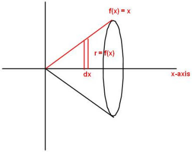


Volumes of solids of revolution

R( ) is the radius from the axis of rotation





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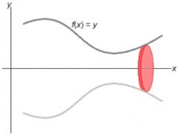

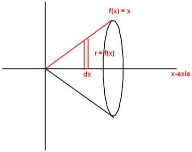

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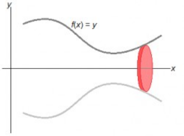

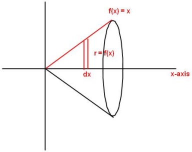

Disk Method (Basic elements are discs)

1. Rotation about the x-axis Volume $= \int_a^b \pi R(x)^2 dx$

Volumes of solids of revolution

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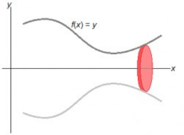

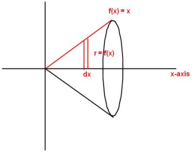

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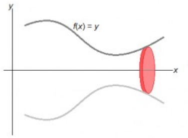

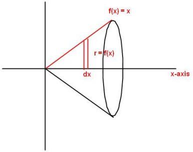

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Washer Method (Basic elements are washers)

$$\text{Volume} = \int_a^b \pi \left[R(x)^2 - r(x)^2 \right] dx$$

Between two curves

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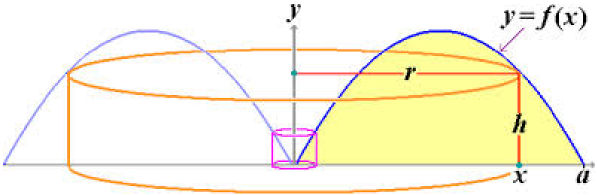
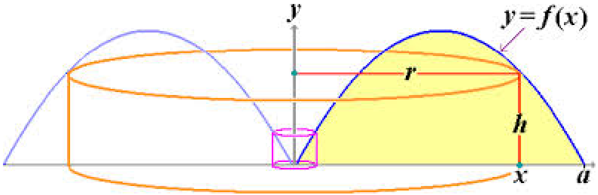
With out cross section



(Basic elements are Shells)

With out cross section

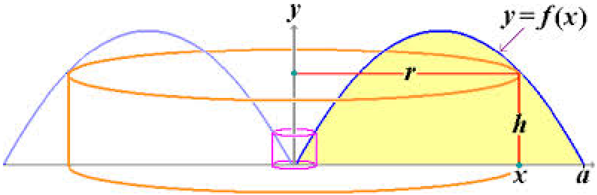


(Basic elements are Shells)

1. Revolving about the y-axis,

$$V = \int_a^b 2\pi(\text{ Shell Radius })(\text{ Shell Height })dx$$

With out cross section



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Note that the formula is in terms of y when revolving about the x-axis and vice versa.

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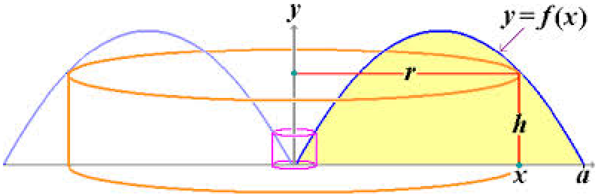

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 Note that the formula is in terms of y when revolving about the x-axis and vice versa.

3. Changing the axis of rotation apart from the standard x-axis