# BM5163 Bayesian Inference in Bioengineering

## Problem Set 2

### Instructions

1. You are expected to work on these problems on your own and not submit the solutions.

### Questions

1. Suppose we are interested in a Poisson process. The prior distribution for $\lambda$ is

$$f(\lambda) = \begin{cases} 0 & \text{for } \lambda \leq 0 \\ c\lambda e^{-\lambda} & \text{for } \lambda > 0 \end{cases}$$

   (a) Find the value of $c$.
   (b) What is the prior mean value of $\lambda$?
   (c) What is the prior standard deviation of $\lambda$?

   We observe data (i.i.d) $x_i$'s for $i \in [1, n]$ from the Poisson distribution.

   (a) Find the likelihood.
   (b) Find the posterior density of $\lambda$.
   (c) What are the posterior mean and standard deviation values of $\lambda$?

2. Repeat the previous question for a binomial distribution with the following prior and a single observation $x$.

$$f(\theta) = c\left(\theta^2(1-\theta) - \theta(1-\theta)^2 + a\right) \text{ for } 0 \leq \theta \leq 1.$$

3. In a medical device calibration process, the output of a device is designed to provide a measurement of a specific physiological parameter, say the blood glucose level. The device is designed to be conservative and does not indicate measurements that are below a certain threshold. Therefore, the remaining measurements may slightly exceed the nominal value. It is believed that the excess value $X$, in mg/dL, has a continuous uniform distribution on $(0, \theta)$ but the value of $\theta$ is unknown. Our prior density for $\theta$ is

$$f(\theta) = \begin{cases} 0 & \text{for } \theta < 0 \\ \dfrac{c}{100} & \text{for } 0 \leq \theta < 10 \\ \dfrac{c}{\theta^2} & \text{for } \theta \geq 10 \end{cases}$$

   (a) Find the value of $c$.
   (b) Find the prior median of $\theta$.

   Suppose, we observe 10 subjects (i.i.d), and their excess BG levels, in mg/dL, are as follows

$$3.8, 2.1, 4.9, 1.8, 1.7, 2.1, 1.4, 3.6, 4.1, 0.8$$

   (a) Find the likelihood.
   (b) Find the posterior density function $h(\theta)$.

4. Suppose we want to know the average TSH levels in the population of a city. For this, we perform measurements in the following manner

(a) blood test to measure TSH levels from 15 randomly chosen individuals

(b) blood test to measure TSH levels from 3 randomly chosen individuals on 5 days

At the end of both measurements, we have 15 values. We seek to use the maximum likelihood approach to identify mean TSH levels. Analyze both scenarios.