

BM5163 Bayesian Inference in Bioengineering

Exam 2

Instructions

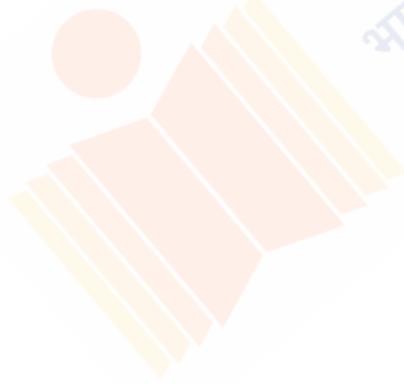
1. This exam is open notes where you can use any hand-written material. Photocopies/prints/books/electronic devices are not permitted to be used.
2. You are expected to work on these problems on your own. **Any reasonable signs of 'copying/plagiarism' will attract penalties.**

Questions

1. In a trial of anti-hypertensive medication, the drop in BP is measured. The measurement setup gives normally distributed data with variance $\sigma^2 = 10 \text{ mmHg}^2$. If the prior distribution for the drop in BP is given by $\mu \sim \mathcal{N}(0, 30)$, answer the following
 - (a) What is the posterior distribution if the trial had 5 patients and an average drop of -10 mmHg was observed in the group? **(20)**
 - (b) Find the minimum number of patients needed in the trial so that the posterior variance drops below 1 mmHg. **(10)**



భారతీయ సాంకేతిక విద్యా సంస్థానం హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad



భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

2. In many biomedical applications, parameters such as glucose levels or biomarker concentrations are estimated from noisy measurements. You are given the following

- Prior distribution

$$\mu \sim \mathcal{N}(100, 10^2)$$

- Measurement model

$$y_i | \mu \sim \mathcal{N}(\mu, 25)$$

and two independent datasets are collected

- $y_1 = 102, y_2 = 98, y_3 = 100$
- $y_1 = 140, y_2 = 145, y_3 = 150$

- Compute the maximum likelihood estimate of μ for both datasets.
- Derive the posterior distribution of μ for each dataset and compute the posterior mean.
- Based on your results, identify which dataset shows a stronger influence of the prior on the posterior estimate.



భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad



భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Rough work

