Last updated on March 25, 2026

BM5163 Bayesian Inference in Bioengineering

Exam 2

Instructions

1. This exam is open notes where you can use any hand-written material. Photocopies/prints/books/electronic devices are not permitted to be used.
2. You are expected to work on these problems on your own. **Any reasonable signs of 'copying/plagiarism' will attract penalties.**

Questions

1. In a trial of anti-hypertensive medication, the drop in BP is measured. The measurement setup gives normally distributed data with variance $\sigma^2 = 10$ mmHg^2. If the prior distribution for the drop in BP is given by $\mu \sim \mathcal{N}(0, 30)$, answer the following
   (a) What is the posterior distribution if the trial had 5 patients and an average drop of -10 mmHg was observed in the group? **(20)**
   (b) Find the minimum number of patients needed in the trial so that the posterior variance drops below 1 mmHg. **(10)**

2. In many biomedical applications, parameters such as glucose levels or biomarker concentrations are estimated from noisy measurements. You are given the following

   - Prior distribution
   $$\mu \sim \mathcal{N}(100, 10^2)$$
   - Measurement model
   $$y_i \mid \mu \sim \mathcal{N}(\mu, 25)$$

   and two independent datasets are collected

   - $y_1 = 102,\ y_2 = 98,\ y_3 = 100$
   - $y_1 = 140,\ y_2 = 145,\ y_3 = 150$

   (a) Compute the maximum likelihood estimate of μ for both datasets.

   (b) Derive the posterior distribution of μ for each dataset and compute the posterior mean.

   (c) Based on your results, identify which dataset shows a stronger influence of the prior on the posterior estimate.

Rough work