Name:
Roll Number:

BM5063 Mathematical Physiology and Systems Medicine

Exam 1 Solution

Instructions

1. In this exam you can use one page of any hand-written material. Photocopies/prints/soft-copies are not allowed.
2. You are expected to answer these on your own. **Any reasonable signs of 'copying/plagiarism' will attract penalties.**
3. You have to provide answers within the space provided. No additional paper will be given.

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1. Consider cortisol production by the adrenal grands that is described by the following system of equations

$$\dot{c} = \alpha_1 A - \beta_1 c$$
$$\dot{A} = \frac{\alpha_2}{K + c^2} - \beta_2 A$$

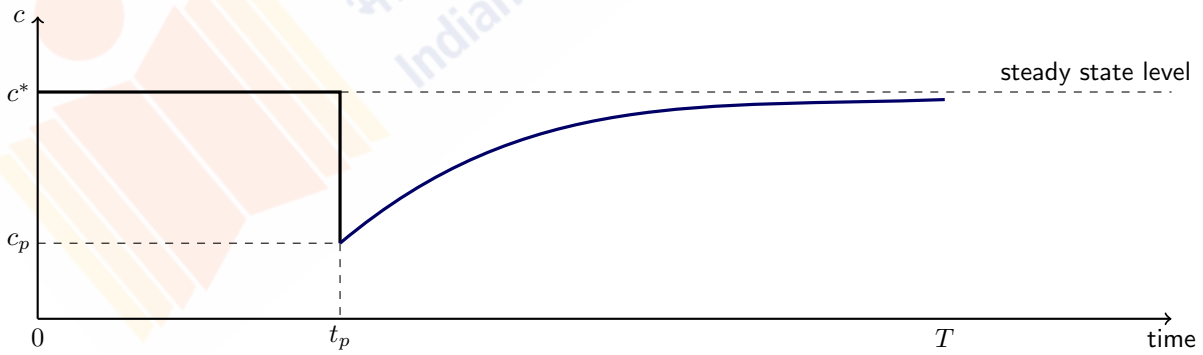
where c and A denote the cortisol levels and the size of the adrenal glands.

(a) Between cortisol and glands dynamics, which of the two is likely to be the faster process and why? **(10)**

Cortisol is a hormone and its dynamics (synthesis, secretion, circulation, and clearance, etc.) take place at short physiological time scales (of the order of minutes). On the other hand, the dynamics of gland size (by cell proliferation and death) takes place at a timescale of the order of days or weeks. Therefore, between the two, cortisol dynamics is a faster process.

(b) Consider that the system is at steady state (c^*, A^*). At time $t = t_p > 0$, the cortisol level suddenly drops to $c_p < c^*$ due to an external perturbation. This drop occurs only once and instantaneously. After this perturbation at $t = t_p$, the system follows the dynamics given by the equations above. Complete the figure below by sketching the curve showing cortisol levels for $t \in [0, T]$, where $T > t_p$. You have to draw the plot in the figure below only. **(10)**

Assuming that the gland size does not change in this duration, we have



(c) Estimate the time at which the cortisol levels are half-way between c^* and c_p after the perturbation. **(10)** Given the faster dynamics of cortisol, ignoring any change in the adrenal gland size, we can assume A to be fixed and solve the $\dot{c}$ equation to get the time for cortisol to reach halfway between c^* and c_p as

$$t_{\text{halfway}} = t_p + \frac{\ln 2}{\beta_1}$$

2. Consider bacterial growth in lungs during an infection. The bacterial population grows through binary fission and the bacteria are killed by the neutrophils. The number of bacteria in the lungs are denoted by b and neutrophil population is denoted by p.

(a) Write down the equation describing the dynamics of bacterial population. **(10)**

There are different ways of writing the equation under different assumptions. Here, one such example solution is given.

$$\dot{b} = \alpha b \left(1 - \frac{b}{K}\right) - \beta p \left(\frac{b}{b_c + b}\right)$$

where K is the carrying capacity of the tissue, and b_c is some critical level of bacterial population.

(b) List all the assumptions behind your answer in part (a). **(15)**

- There is a finite space and nutrient supply for bacterial sustenance. Therefore K.
- Neutrophils do not affect the bacterial fission process.
- The bacterial death is only via neutrophils.
- The neutrophil population is smaller than that of bacteria. Therefore, saturation in the second term.