BM5063 Mathematical Phyiology and Systems Medicine
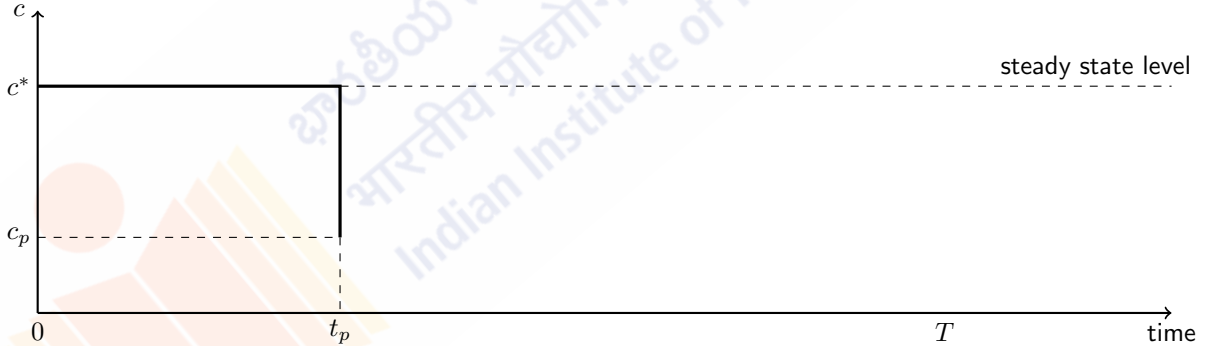
Exam 1

Instructions

1. In this exam you can use one page of any hand-written material. Photocopies/prints/soft-copies are not allowed.

2. You are expected to answer these on your own. **Any reasonable signs of 'copying/plagiarism' will attract penalties.**

3. You have to provide answers within the space provided. No additional paper will be given.

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1. Consider cortisol production by the adrenal grands that is described by the following system of equations

$$\dot{c} = \alpha_1 A - \beta_1 c$$
$$\dot{A} = \frac{\alpha_2}{K + c^2} - \beta_2 A$$

where c and A denote the cortisol levels and the size of the adrenal glands.

(a) Between cortisol and glands dynamics, which of the two is likely to be the faster process and why? **(10)**

(b) Consider that the system is at steady state (c^*, A^*). At time $t = t_p > 0$, the cortisol level suddenly drops to $c_p < c^*$ due to an external perturbation. This drop occurs only once and instantaneously. After this perturbation at $t = t_p$, the system follows the dynamics given by the equations above. Complete the figure below by sketching the curve showing cortisol levels for $t \in [0, T]$, where $T > t_p$. You have to draw the plot in the figure below only. **(10)**

(c) Estimate the time at which the cortisol levels are half-way between c^* and c_p after the perturbation. **(10)**

2. Consider bacterial growth in lungs during an infection. The bacterial population grows through binary fission and the bacteria are killed by the neutrophils. The number of bacteria in the lungs are denoted by b and neutrophil population is denoted by p.

   (a) Write down the equation describing the dynamics of bacterial population. **(10)**

   (b) List all the assumptions behind your answer in part (a). **(15)**