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BM5063 Mathematical Physiology and Systems Medicine

Problem Set 2

Instructions

1. You are not expected to submit answers to these problems

Questions

1. Analyze the following systems, identify FPs, and their stability properties using the phase-plane method and by the method of perturbation
   (a) $\dot{x} = \frac{3y^2}{1+y^2} - x,\ \dot{y} = \frac{3x^2}{1+x^2} - y$
   (b) $\dot{x} = \frac{4}{1+y^2} - x,\ \dot{y} = \frac{4}{1+x^2} - y$
   (c) $\dot{x} = x - xy,\ \dot{y} = 2xy - y$
2. In the following systems, write down equations describing system dynamics. Identify fixed points and analyze the stability of the fixed points.
   (a) Consider two genes X and Y. X is activated by its own protein product and repressed by the protein product of gene Y. Y is activated by the protein product of X.
   (b) Two drugs D_1 and D_2 interact in the human body in a way that each enhances the clearance of the other from the system. What happens when one drug is withdrawn?
   (c) Immune cells C secrete a cytokine y. The cytokine promotes immune cell proliferation. Both decay naturally.
   (d) Pathogens P grow by fission (exponentially). Immune cells I kill the pathogens and also proliferate in the presence of P.
   (e) Two transcription factors repress each other.
3. Cells have clocks that track the time of day called circadian clocks. Beta cells secrete more insulin for a given level of glucose during the day than during the night. Modify the model discussed in the class to reflect this effect.
   - What does the new model predict for steady-state glucose and insulin during the day and night?
   - Is it unhealthy to eat big late-night meals?
4. The brain takes up glucose from the blood at an insulin-independent rate.
   (a) Modify the glucose-insulin-Beta cell model to describe this effect.
   (b) Is the steady-state blood glucose level affected by the brain's uptake rate?
5. As discussed in the class, consider the carrying capacity of the Beta cells where their growth cannot go on beyond a limit. You can use logistic growth to model this effect.
   (a) Write down the equations.
   (b) How many fixed points are there in this system? Interpret these fixed points in terms of healthy and diseased states.
   (c) Write `Python` code to simulate this effect and show it can result in pre-diabetes.
6. Using the `Python` code for Glucose-Insulin-Beta cells model discussed in the class analyze the following
   (a) Onset of prediabetes where blood glucose levels reach a higher value due to a saturation in Beta-cells mass. Identify which factors can cause prediabetes.

(b) Consider a drop in insulin sensitivity under two conditions- (i) sudden drop, and (ii) gradual drop. Which one is more likely to result in prediabetes?

(c) Consider two scenarios- (i) a small drop in insulin sensitivity over a long duration, and (ii) a large drop in insulin sensitivity over a small duration. Which is more likely to cause T2 diabetes?

(d) Analyze the role of food intake on the onset of diabetes.