Last updated on January 20, 2026

BM5063 Systems Medicine

Problem Set 1

Instructions

1. You are not expected to submit answers to these problems

Questions

1. Analyze the following systems, identify FPs, and their stability properties using the phase-plane method and by the method of perturbation
   (a) $\dot{x} = -x$
   (b) $\dot{x} = -x \pm x^2$
   (c) $\dot{x} = -x \pm x^3$
   (d) $\dot{x} = -x + \dfrac{x}{k+x}$
   (e) $\dot{x} = -x + \dfrac{k}{k+x}$
   (f) $\dot{x} = -x + \dfrac{x^2}{k^2+x^2}$
2. Use computer (for example, you can use https://www.desmos.com/calculator) to draw phase diagram and identify the nature of stability of the fixed point in the following systems
   (a) $\dot{x} = -x + \dfrac{x^4}{k^4+x^4}$
   (b) $\dot{x} = -x + \dfrac{k^4}{k^4+x^4}$
3. For the following problems, sketch the curve marking the fixed points as a function of parameter a.
   (a) $\dot{x} = -ax$ for $a \in [-1, 1]$
   (b) $\dot{x} = -x + \dfrac{ax^2}{1+x^2}$ for $a \geq 0$
   (c) $\dot{x} = -x + \dfrac{a}{1+x^2}$ for $a \geq 0$
4. Analyze the following scenarios
   (a) Linear clearance in drug pharmacokinetics
   $$\dot{c} = I - kc$$
   where I is the fixed drug input rate and the second term is the drug clearance rate.
   (b) Tumor volume growth
   $$\dot{v} = av \log\left(\frac{v_c}{v}\right)$$
   where a and v_c are two constants. This model is known in *Gompertz growth model*.

5. In a healthy immune response, inflammation (I) is triggered by a pathogen and then resolved by anti-inflammatory mechanisms. In some severe cases (like some viral infections), inflammation can become self-sustaining. It is called a "cytokine storm". Consider a simplified 1D model where the pathogen load is treated as a constant p, and the immune system has a positive feedback mechanism (self-recruitment of leukocytes)

$$\dot{I} = p - I + \frac{aI^2}{1 + I^2}$$

where a is a parameter.

- In the absence of any pathogen $p = 0$, what is/are the healthy steady states of the system? Which is the healthy steady state?
- As the pathogen stimulus p increases, how does the number of fixed points change?
- What happens to the patient if the stimulus p pushes the system past the unstable fixed point, so that, even if the pathogen is later removed (p returns to 0)? Is the inflammation reversible or irreversible?