Last updated on April 8, 2025

BM4040 Mechanobiology

Problem set 6

Instructions

- You are not expected to submit answers to these problems.

Questions

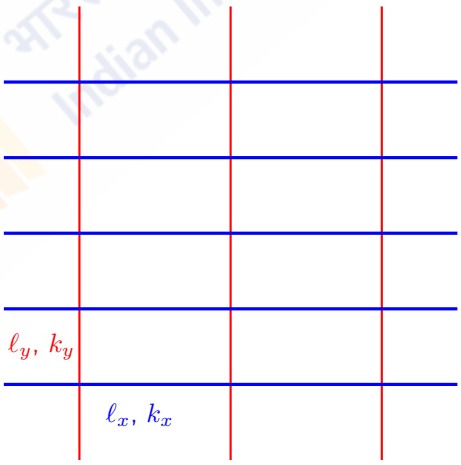
1. For networks with four-fold symmetry we have seen that the free energy density is given by

$$U = \frac{K_a}{2}\left(\epsilon_{xx} + \epsilon_{yy}\right)^2 + \frac{\mu_p}{2}\left(\epsilon_{xx} - \epsilon_{yy}\right)^2 + 2\mu_s\epsilon_{xy}^2.$$

   By minimizing enthalpy of the network (microscopic perspective) show that for a network under isotropic tension τ the elastic constants are given by

$$K_a = \frac{k_s - \tau}{2}$$
$$\mu_p = \frac{k_s + \tau}{2}$$
$$\mu_a = \tau$$

2. The spring network with four-fold symmetry (as in the last question) can be generalized to a rectangular network of two spring types (two-fold symmetry), as shown in the figure below



   (a) Derive the expression for macroscopic free energy density.

   (b) Prove that, once equilibrated at a specific tension τ the strain in the x direction is given by

$$\epsilon_{xx} = \frac{\tau\ell_y k_y + \tau^2 l_x}{k_x k_y - \tau^2}$$

   (c) Calculate the strain in the y direction under tension τ.

   (d) To first order in τ, what is the area per vertex as a function of tension?

3. In class we looked at the network with six-fold symmetry (without applied tension) and compared the macroscopic energy density (from the continuum model) and microscopic energy density by writing down the energy stored in each spring. Using symmetry arguments we have concluded that for six-fold symmetric networks, the macroscopic energy can be described using two elastic coefficients K_a and μ

$$U = \frac{K_a}{2}\left(\epsilon_{xx} + \epsilon_{yy}\right)^2 + \mu\left[\frac{\left(\epsilon_{xx} - \epsilon_{yy}\right)^2}{2} + 2\epsilon_{xy}^2\right]$$

Now consider a network with the same topology but it is composed of three types of filaments (with stiffness k_1, k_2, and k_3) as shown in the figure below.

(a) Using the symmetry arguments write down the macroscopic energy density expression in terms of the elastic constants and strains. Enumerate your rationale.

(b) Now consider the case where two filaments are of the same type and the third one is different. Write down the macroscopic energy density expression.

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