Last updated on March 2, 2025

BM4040 Mechanobiology

Problem set 6

Instructions

- You are not expected to submit answers to these problems.

Questions

1. Complete the derivation of the persistence length to show

$$l_p = \frac{EI}{k_BT}.$$

Work out all the steps missed in the class. You can start with

$$p(\theta) = \frac{1}{Z} e^{-U(\theta)/k_BT}$$

where

$$Z = \int_0^{2\pi}\int_0^{\pi} e^{-U(\theta)/k_BT} d\phi \sin\theta d\theta$$

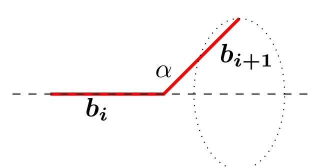
where integration is with respect to the solid angle $d\phi \sin\theta d\theta$ in 3D. From this, you can calculate

$$\langle\theta^2\rangle = \frac{1}{Z}\int_0^{2\pi}\int_0^{\pi} \theta^2 e^{-U(\theta)/k_BT} d\phi \sin\theta d\theta = 2\frac{k_BT}{EI}$$

2. Consider a large motor neuron running from the brain to the arm containing a core bundle of microtubules. Taking the persistence length of a microtubule to be 2 mm, what energy is required (in k_BT at 300 K) to bend a microtubule of length 20 cm into an arc of radius 10 cm?

3. Show that the curvature C of the trajectory of a particle moving with velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ can be found from the cross product $|\boldsymbol{v} \times \boldsymbol{a}| = Cv^3$.

4. Consider a polymer, where the bond angle between successive carbon atoms is a fixed value α, although the bonds are free to rotate around one another.



The length and orientation of the bond between node i and atom $i+1$ define a bond vector $\boldsymbol{b_i}$. Assume all bond lengths are the same, and that remote bonds can intersect.

(a) Show that the average projection of $\boldsymbol{b}_{i+k}$ on $\boldsymbol{b}_i$ is

$$\langle \boldsymbol{b}_{i+k} \cdot \boldsymbol{b}_i \rangle = b^2 \left(-\cos\alpha\right)^k$$

(b) Write $\langle \boldsymbol{R}^2 \rangle$ in terms of $\langle \boldsymbol{b}_i \cdot \boldsymbol{b}_j \rangle$ to obtain

$$\frac{\langle \boldsymbol{R}^2 \rangle}{b^2} = N \left[1 + \left(2 - \frac{2}{N} \right) (-\cos\alpha) + \left(2 - \frac{4}{N} \right) (-\cos\alpha)^2 + \cdots \right].$$

(c) For large N show that

$$\langle \boldsymbol{R}^2 \rangle = N b^2 \left(\frac{1 - \cos\alpha}{1 + \cos\alpha} \right)$$