BM4040 Mechanobiology

Problem set 3

Instructions

- You are not expected to submit answers to these problems.

Questions

1. Consider a strand of DNA that only contains two different nucleotides, A or T. Using the binomial coefficient, calculate the number of different possible sequences if the strand is N nucleotides long, and only n of the nucleotides are T.

2. Consider a molecule that has a "home" position and a linear restoring force that is proportional to the distance that it is away from this position. In other words, the molecule acts like it is attached to the tip of a spring. The restoring force is $f = -kx$, where x is the distance from the home position and k is the spring constant. The potential energy is a function of position and can be expressed as

$$U(x) = \frac{1}{2}kx^2.$$

   We can treat each molecular position of the molecule, x, as a microstate of the system. In this case, assuming thermal equilibrium, the probability of finding the molecule in any given position is given by the Boltzmann distribution. What is the average energy?

3. Consider a two-dimensional model polymer composed of $n + 1$ rigid segments, with neighboring segments connected by a rotating joint with a rotational spring. The rotational spring at each joint can be in three discrete states- 0° (hairpin), 90° (right angle), or 180° (straight links). Assume that these three states cost ϵ_1, ϵ_2, and 0 energy, respectively, with $\epsilon_1 > \epsilon_2$. Estimate the equilibrium end-to-end length of the chain at temperature T. You can first test with small values of n and then generalize for large n.

   Health warning: *This exercise may not be a straightforward classroom problem.*