BM4040 Mechanobiology

Problem set 1

Instructions

- You are not expected to submit answers to these problems.

Questions

1. A cylinder of length L and radius R is a composite of two materials, with elastic constants (E_1, ν_1, G_1) and (E_2, ν_2, G_2). The two materials span the cylinder for $0 \le r < R_c$ and $R_c \le r < R$, respectively. The cylinder is axially loaded with a pressure of magnitude P.

   (a) Write down equations describing mechanical equilibrium and corresponding boundary conditions.

   (b) Estimate stress components.

   (c) What are the length and radius of the cylinder in the deformed configuration?

2. Consider the same scenario as in the previous question but the two materials are spanning the cylinder as $0 \le z < L_c$ and $L_c \le z < L$.

3. A cylinder of length L and radius R is made up of a material of density ρ_c. It is kept vertically floating in a fluid of density ρ.

   (a) Write down equations describing mechanical equilibrium and corresponding boundary conditions.

   (b) Estimate stress components.

   (c) What are the length and radius of the cylinder in the deformed configuration?

4. Consider a cube-shaped block of edge length l made up of a material that ruptures if shear stress crosses a critical value τ. This block is kept on a rigid substrate and its top surface is pushed (uniformly) with a normal force f_n and a tangential force f_t along the x-direction. Identify the plane along which the block is likely to rupture.

5. Consider an elastic rod of length L with a square cross-section of side length a. The rod is placed along the x-axis with on end at $x = 0$ and the other at $x = L$. The Young's modulus of the rod is position dependent as is given by

$$E(x) = E_0 + \gamma \frac{x}{L} \left(1 - \frac{x}{L}\right) \tag{1}$$

   where *gamma* is a constant. Other elastic parameters are independent of x. The $x = 0$ end of the rod is held fixed and the other end is applied by a tensile force f.

   (a) Estimate the x component of the displacement in the rod as a function of x.

   (b) What is the length of the rod at equilibrium?

   (c) Does the cross-sectional area of the rod change when this force is applied? Estimate the change.