BM2053 Mathematical Models & Systems Biology

Problem Set 3

Instructions

1. You are not supposed to submit the solutions to these questions.
2. To get the most out of these, solve all the problems on your own.

Questions

1. Consider a protein Y under positive autoregulation, that is Y↻. If the equation describing the concentration of the protein is as following
$$\frac{dy}{dt} = \alpha + \alpha_1 y - \beta y.$$
with $\alpha > 0$, $\beta > 0$ and $\alpha_1 > 0$. Here we have replaced Hill function with $\alpha + \alpha_1 y$.
   (a) Draw the phase diagram.
   (b) Identify the fixed points from the phase plot and write down their stability properties.
   (c) Solve the above equation analytically and check if the steady states obtained after solving the equation match the ones obtained from the rate plot.
   (d) Non-dimensionalize the equation.
   (e) Solve the equation numerically.
   (f) Does this system show qualitative change in behavior? If yes, identify the nature of the bifurcation.

2. Consider a protein X under negative autoregulation, that is X↻. Imagine that the protein expression is at the steady state and suddenly the maximal promoter activity is reduced to $\alpha = 0$.
   (a) Write down the equation describing the dynamics after this sudden change.
   (b) What is the response time in this scenario.

3. In a transcription network protein A activates X. X suppresses another gene Y. X activates another gene Z. Protein Y activates A.
   (a) Draw the network using the →, ⊣ notation.
   (b) Write down the equation describing the rate of change in the concentrations of all the proteins mentioned above.
   (c) Write a `Python` program to solve the system of equations.

4. In a transcription network protein A activates another one, B. On the other hand B represses A.
   (a) Draw the network using the →, ⊣ notation.
   (b) Write down the equation describing the concentrations of A and B.
   (c) Assume activation and repression coefficients in the Hill function to be $K = 1\mu$M, $n = 1$, $\alpha = 1\mu$M per sec, $\beta = 0.5$ per sec and obtain the concentrations of A and B at the steady state.
   (d) Solve the equations numerically and plot the concentrations as a function of time. You can take initial concentrations of A and B to be zero.