BM2053 Mathematical Models & Systems Biology

Problem Set 2

Instructions

1. You are not supposed to submit the solutions to these questions.
2. To get the most out of these, solve all the problems on your own.

Questions

1. For the following systems, find fixed points, determine their stability, sketch the bifurcation diagram (x^*, μ), and identify the type of bifurcation.
   (a) $\dot{x} = \mu x - x^2 + x^3$
   (b) $\dot{x} = x(\mu - x) - x^2$
   (c) $\dot{x} = \mu - x + x$
   (d) $\dot{x} = \mu x \left(1 - \frac{x}{K}\right) - 2$
   (e) $\dot{x} = \mu x - x^3 + x^5$
   (f) $\dot{x} = \mu x + x^3 - x^5$
   (g) $\dot{x} = \mu x - xy$, $\dot{y} = -y + x^2$
   (h) $\dot{x} = x\left(1 - \frac{x}{\mu}\right) - \frac{x}{1+x}$
   (i) $\dot{x} = \mu x - x^3 + \epsilon x^2$, where $0 < \epsilon \ll 1$
   (j) $\dot{x} = \mu x \left(1 - \frac{x}{2}\right)\left(\frac{x}{A} - 1\right)$. In this, both μ and A are control parameters.
   (k) $\dot{x} = x\left(1 - \frac{x}{2}\right) - xy$, $\dot{y} = \mu xy - y$
   (l) $\dot{x} = x(1 - x - y)$, $\dot{y} = y(\mu - x - y)$
   (m) $\ddot{y} = \mu\dot{y} - \dot{x}^3 - y$
   (n) $\dot{x} = \mu - x^2 - y$, $\dot{y} = x - y$
   (o) $\dot{x} = \mu x - x^3 - y$, $\dot{y} = \mu y - y^3 - x$
2. For the following non-dimensionalize the equations and identify the system behavior.
   (a) A population $x(t)$ follows logistic growth but the growth rate is reduced are low density due to cooperative effects. The growth rate is given by
   $$\dot{x} = rx\left(1 - \frac{x}{K}\right)\left(\frac{x}{A} - 1\right)$$
   (b) Two species X and Y compete for a shared resource. Their dynamics is given by
   $$\dot{x} = rx\left(1 - x - ay\right)$$
   $$\dot{y} = sx\left(1 - bx - y\right)$$

(c) In a biochemical reaction, a metabolite follows the dynamics

$$\dot{x} = k(\alpha - x) - k_2 x^2$$

(d) A simplified model of neuron firing is given by

$$\dot{v} = v(\mu - v)(v - \alpha)$$

(e) Following are the equations for predator-prey dynamics in some ecosystem

$$\dot{x} = rx\left(1 - \frac{x}{K}\right) - \frac{axy}{1 + bx}$$
$$\dot{y} = cy\left(1 - \frac{y}{gx}\right)$$

(f) In one gene regulatory network, two genes affect each other's dynamics in the following manner

$$\dot{x} = \mu - x - xy^2$$
$$\dot{y} = \mu - y - yx^2$$

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