Name: _____

Roll No.: _____

# BM2053 Mathematical Models & Systems Biology

## Exam 2 Solution sketch

**Duration: 45mins** **Total marks: 40**

1. Write the answers very clearly and within the answer sheets provided. **No additional sheet of paper will be accepted.**
2. Do not use a pencil to write answers or sketch the curves.

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1. In a simplified model of immune regulation, let $x(t) \geq 0$ denote the normalized population of activated immune cells and $y(t) \geq 0$ denote the concentration of an inhibitory cytokine. Activated immune cells promote their own expansion when the net activation drive is positive, while crowding and cytokine-mediated inhibition limit their growth. Consider the nondimensional system

$$\frac{dx}{dt} = x(\mu - x - y),$$
$$\frac{dy}{dt} = x - y,$$

where $\mu$ is a parameter representing the net immune activation drive, and answer the following

(a) Determine the fixed points of the system as functions of $\mu$. **(6)**
The fixed points are $(x^*, y^*) = (0, 0)$ and $(\mu/2, \mu/2)$.

(b) Identify the nature of the stability of the fixed points. **(12)**
The Jacobian at the fixed points is given by

$$J = \begin{bmatrix} \mu - 2x^* - y^* & -x^* \\ 1 & -1 \end{bmatrix} \tag{1}$$

which has eigenvalues

i. $\lambda = -1$ and $\mu$ for $(0, 0)$ fixed point $\Rightarrow$ stable only if $\mu < 0$.

ii. $\lambda = \dfrac{-(2+\mu) \pm \sqrt{(2+\mu)^2 - 16\mu}}{4}$ for the $(\mu/2, \mu/2)$ fixed point. $\Rightarrow \mathbb{R}(\lambda) < 0 \Rightarrow$ stable

(c) Construct the bifurcation diagram and identify the bifurcation, if any. **(12)**
Transcritical bifurcation at $\mu = 0$

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**Rough work**