BM2000 Control Systems

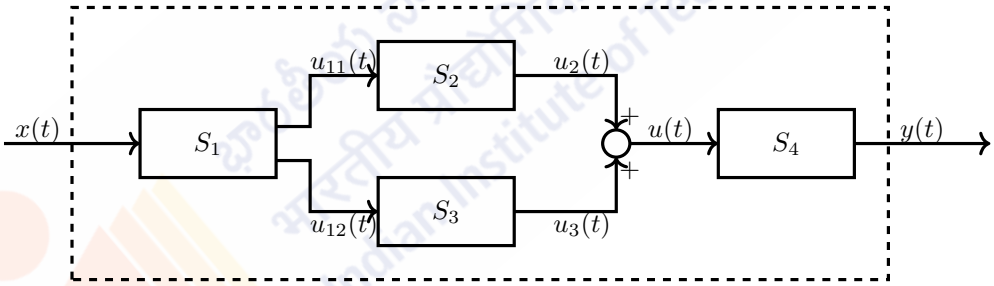
Problem set 1

Instructions

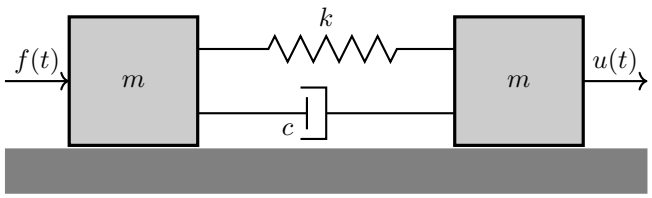
- You are not expected to submit the answers to these questions.

Questions

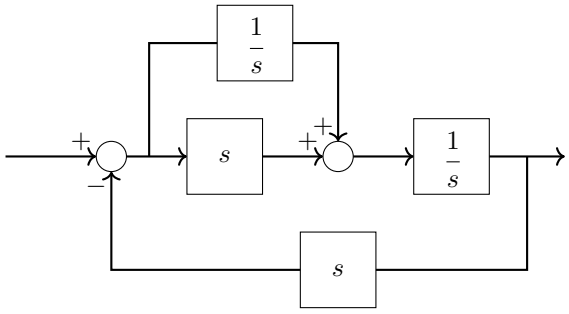
1. Calculate the transfer functions for the following systems and identify the poles and zeros.
   (a) $\ddot{y}(t) + \dot{y}(t) - 2y(t) = \dot{x}(t) - x(t)$
   (b) $\dddot{y}(t) + \ddot{y}(t) + \dot{y}(t) + y(t) = x(t)$
   (c) $\ddot{y}(t) - \dot{y}(t) = \ddot{x}(t)$
2. Calculate the unit impulse and unit step responses for the above systems.
3. Check the bounded input, bounded output (BIBO) stability of these systems.
4. Consider a system (marked with dashed box) which is composed of subsystems S_1, S_2, S_3 and S_4, as shown in the block diagram below.



   The transfer functions corresponding to the two outputs of subsystem S_1 are given by $G_{11}(s)$ and $G_{12}(s)$, and the transfer functions for the subsystems S_2, S_3 and S_4 are $G_2(s)$, $G_3(s)$ and $G_4(s)$, respectively. At the summing junction the two outputs combine as $u(t) = u_2(t) + u_3(t)$. With this information derive the transfer function for this systems as a whole with $x(t)$ and $y(t)$ as input and output, respectively.
5. Consider the following mechanical system consisting of two identical blocks of mass m on a frictionless surface, connected with a spring (spring constant k) and a dashpot with coefficient c.



   A force $f(t)$ (*system input*) is applied on the left block of mass and the displacement of the right mass $u(t)$ (*system output*) is measured as a function of time. Obtain the transfer function $G(s) = \mathcal{L}[u(t)]/\mathcal{L}[f(t)]$ for this system.
6. Identify if the following system is BIBO stable.



7. In this question you are to determine if some systems are linear and time-invariant using numerical tests. For this download the file located here, unzip it and follow the following steps

   (a) Create a Python script similar to the following (put the correct location in place of `<LOCATION OF THE UNZIPPED FILE>`)

```
import sys
import numpy as np
import matplotlib.pyplot as plt

sys.path.append('<LOCATION OF THE UNZIPPED FILE>')

import LTI

T = 20 # total time
t = np.linspace(0,T,1000) # time stamps for system observation

# unit step input,  starting at t = 2
ut = 0*t; ut[t>2] = 1.0

# unit step input,  starting at t = 0
ut = 0*t

yt = LTI.system1(t,ut) # for system 1
#yt = LTI.system2(t,ut) # for system 2
#yt = LTI.system3(t,ut) # for system 3
#yt = LTI.system4(t,ut) # for system 4

plt.plot(t,ut,'r'); # plotting system input
plt.plot(t,yt,'g'); # plotting system output
plt.show()
```

   (b) This question contains 4 systems which you can access by using `LTI.system1(t,ut)` or `LTI.system2(t,ut)` or `LTI.system3(t,ut)` or `LTI.system4(t,ut)` as shown in the code above. These functions return the system output for the given input `ut`. You can take any input and pass it to these functions and obtain the outputs of the four systems. In the code above the unit step input is shown just as an example.

   (c) By choosing appropriate inputs for the systems and tests you have to identify which of the four systems given in `LTI` module are linear and time invariant

   (d) You do not have to submit the code you used to reach the answer. But **you have to describe the inputs you chose, output obtained and how did you reach the final conclusion.**

8. In this question you are to determine the transfer function of some systems using numerical tests. In this question the four systems are either of first order or second. You can use the same script as above but with a small change that instead of `import LTI` you have to use `import SysCharac`.

   (a) Following the same steps as in previous questions you have to obtain the system outputs for four systems (to be obtained by `SysCharac.system1(t,ut)` etc.) for **appropriate inputs**.

   (b) In order to identify the system you have to extract the zeros and poles of the four systems described in the file.

   (c) Here also you do not have to submit the code you used to reach the answer. But **you have to describe the inputs you chose, output obtained and how did you reach the final conclusion.**