Parameterized Approx-Scheme for Independent Set of Rectangles

Fahad Panolan

Department of Computer Science & Engineering
IIT Hyderabad

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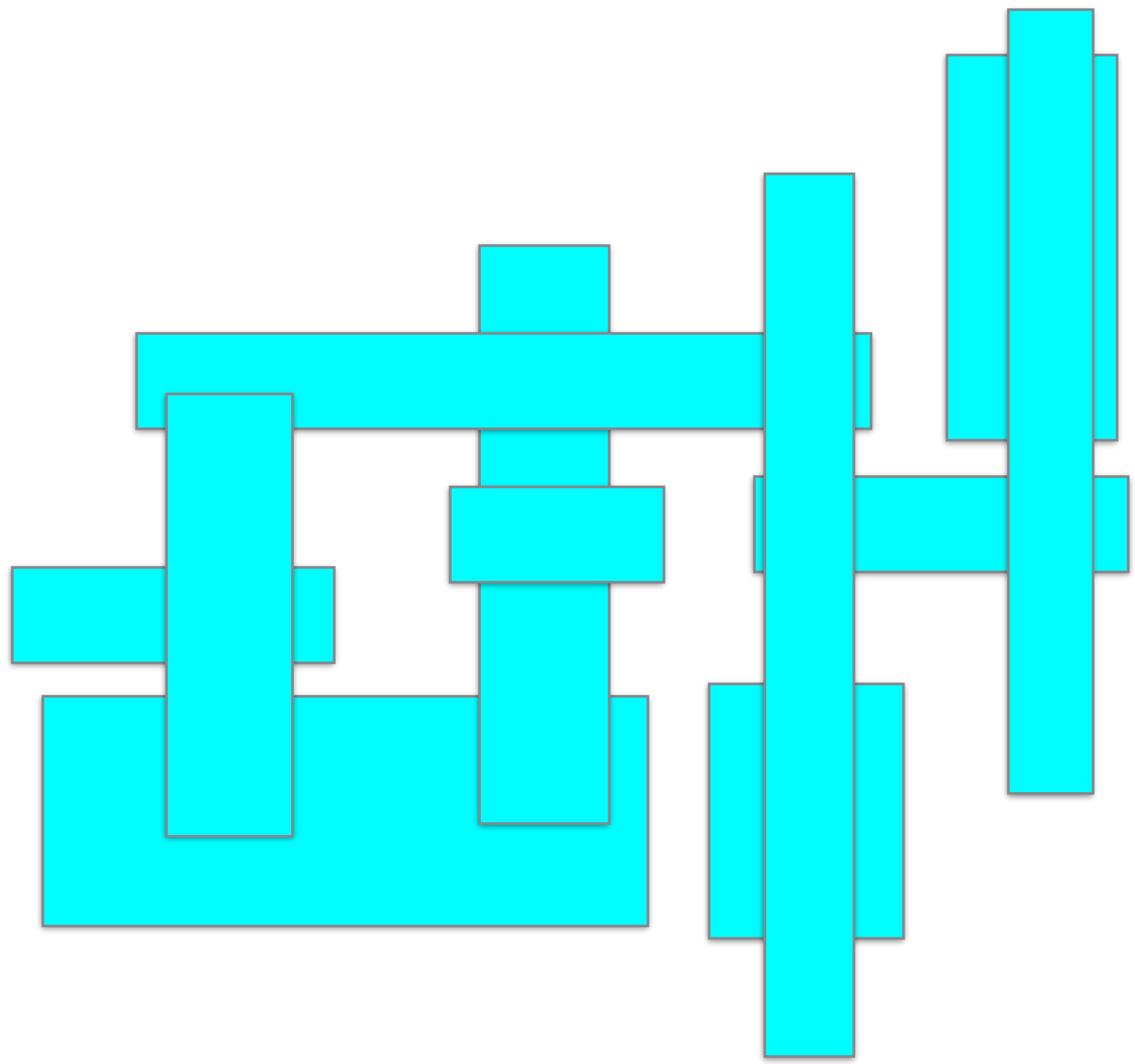
Outline

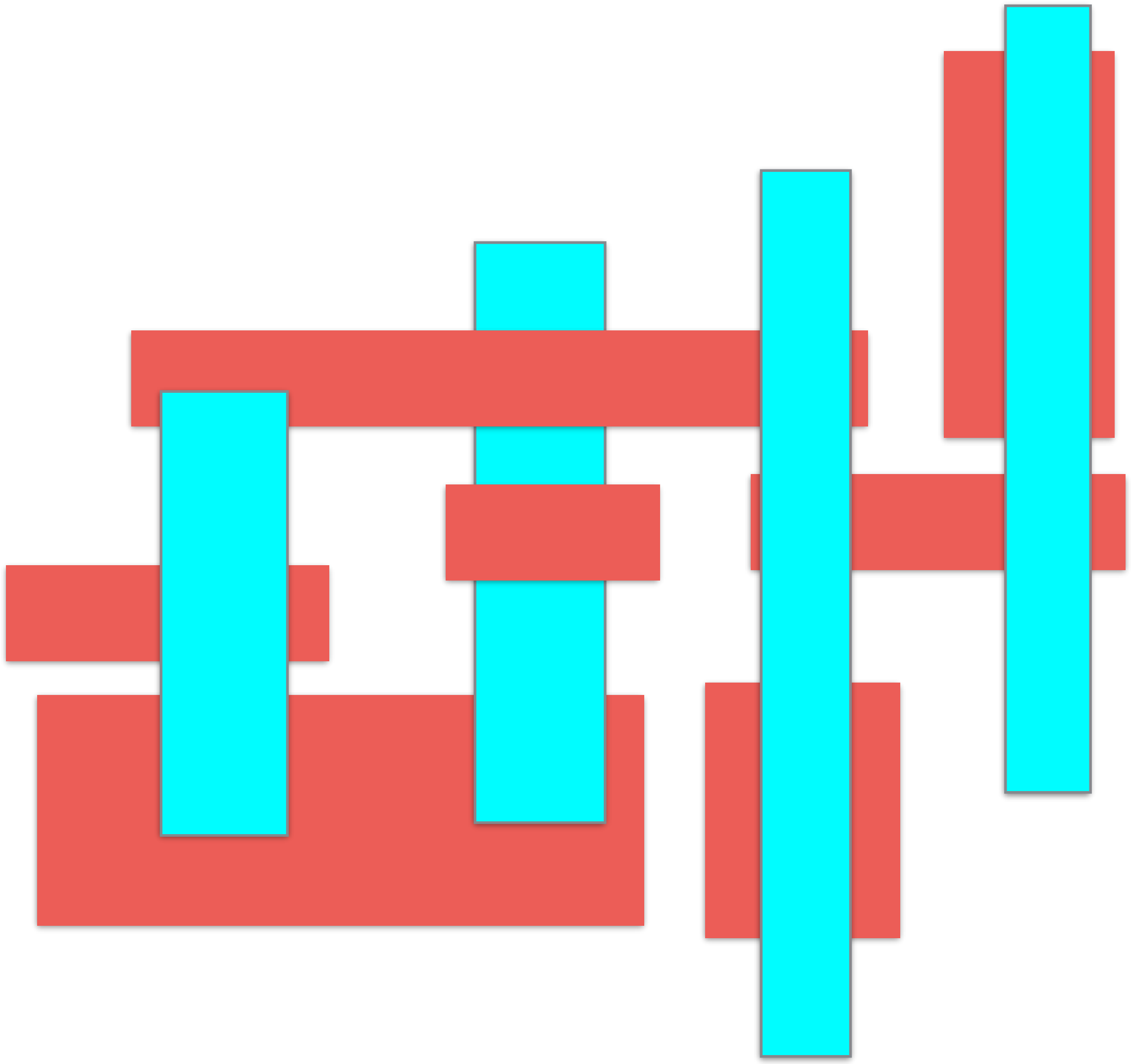
- One problem
- One algorithm
- One open problem

Independent Set of Rectangles



and an integer k

Independent Set of Rectangles



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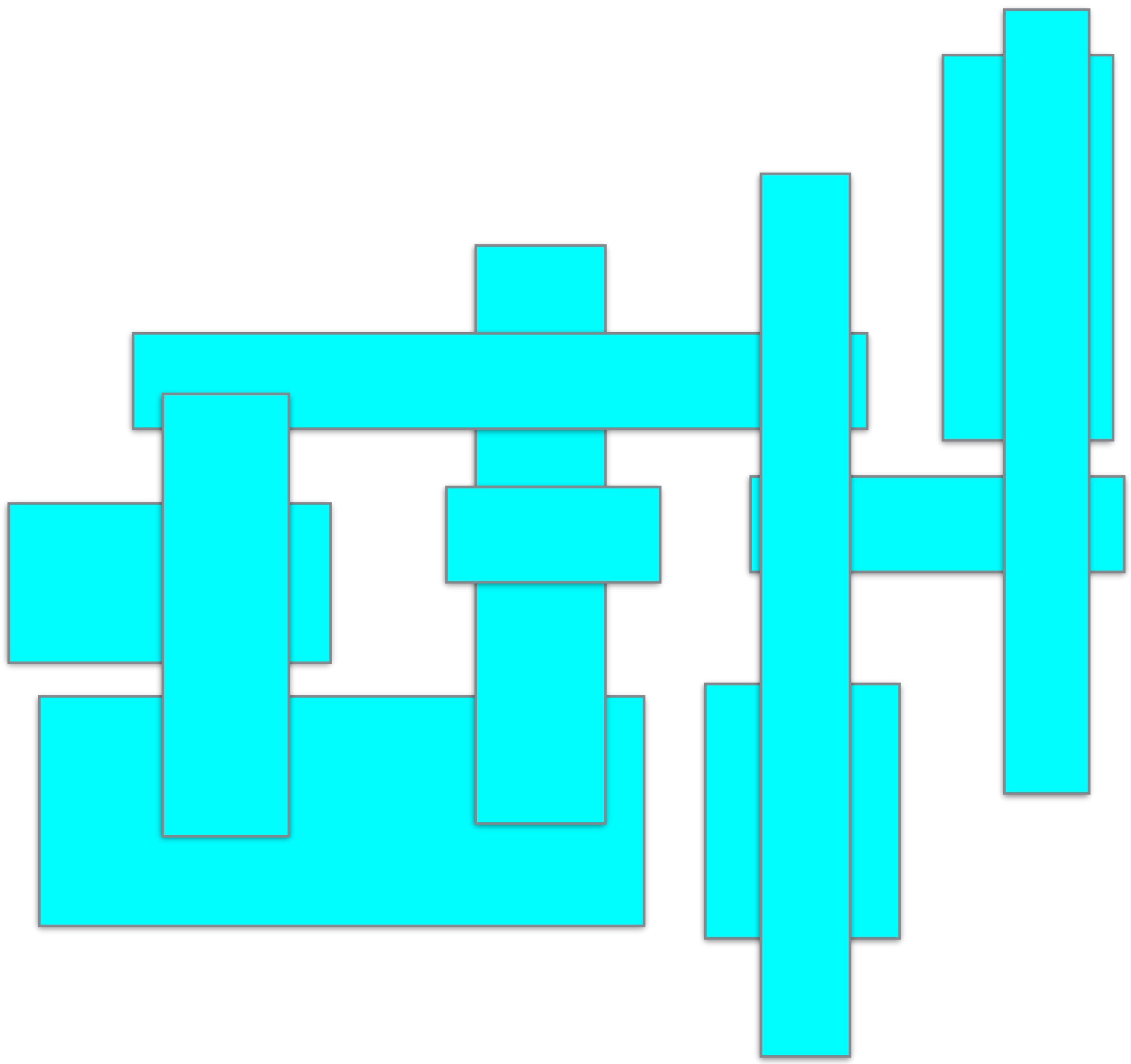
Independent Set of Rectangles

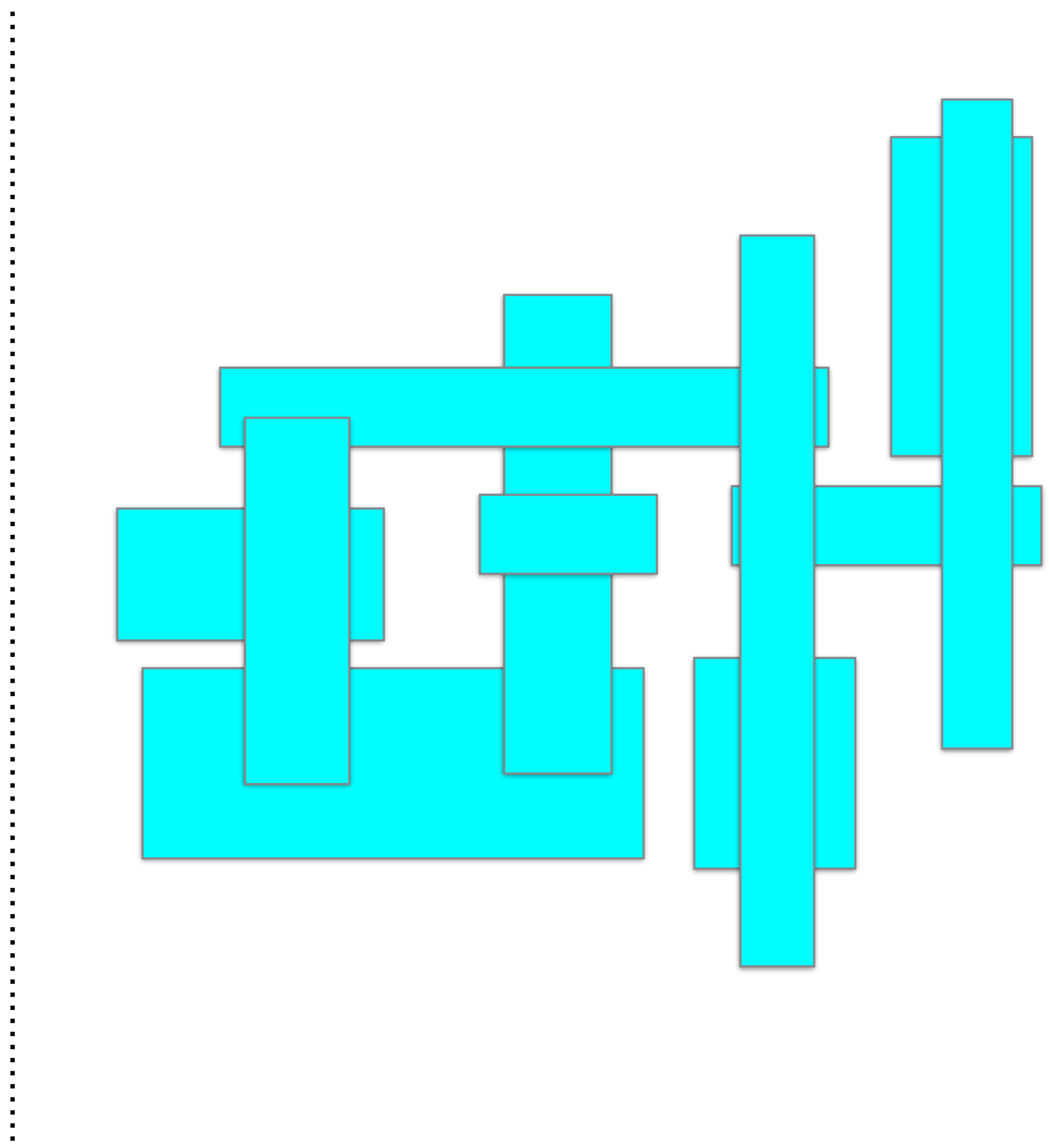
- QPTAS [Adamaszek and Wiese, 2013]
- O(loglog n)-Approx [Chalermsook and Chuzhoy, 2009]
- W[1]-Hard [Marx, 2005]
- PAS [Grandoni et al., 2019]

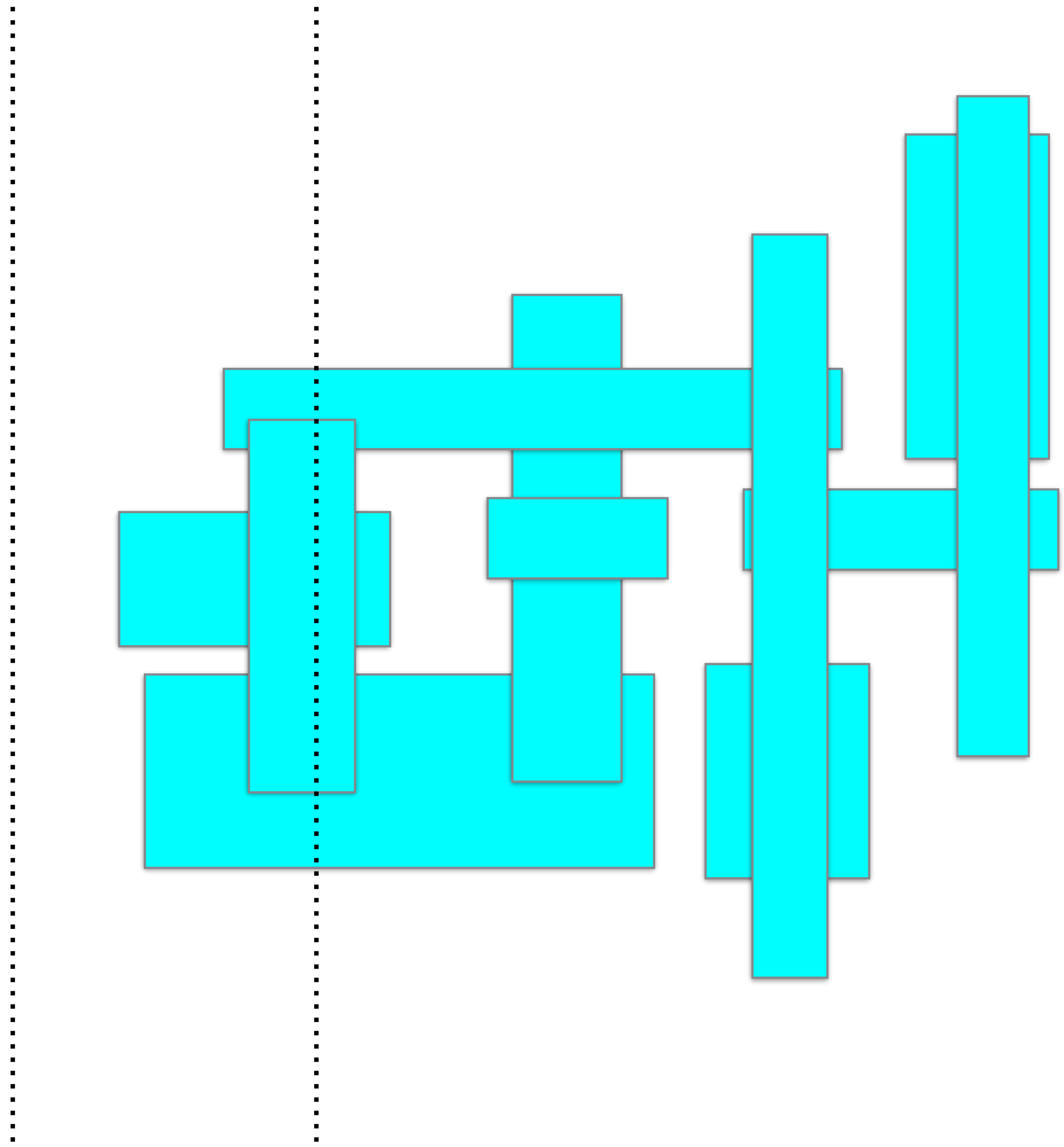
For any $\varepsilon>0$, there is an algorithm

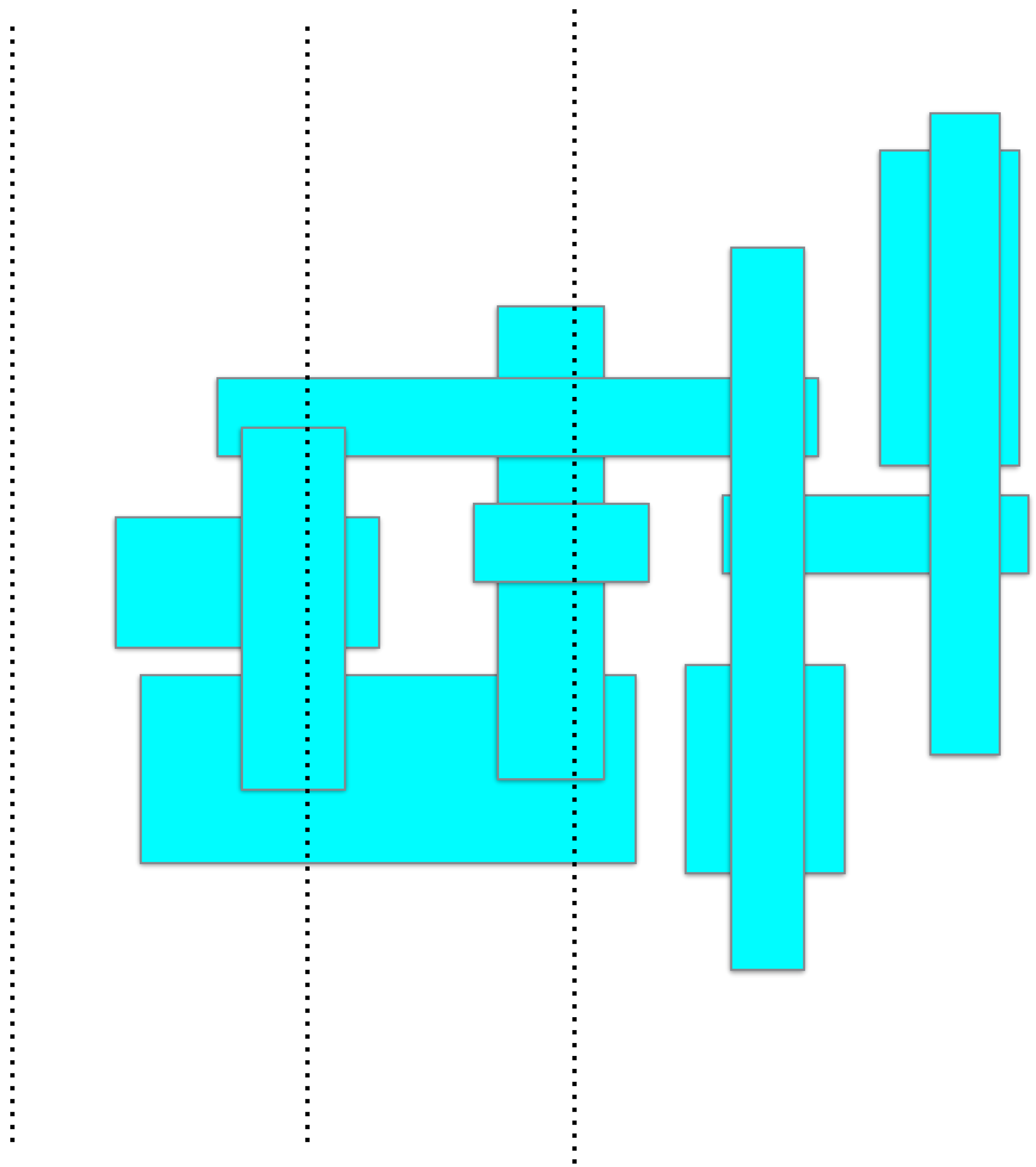
- running in time: $f(k,\varepsilon)\text{poly}(n)$
- outputs a set of $(1-\varepsilon)\min(k,\text{opt})$ independent rectangles

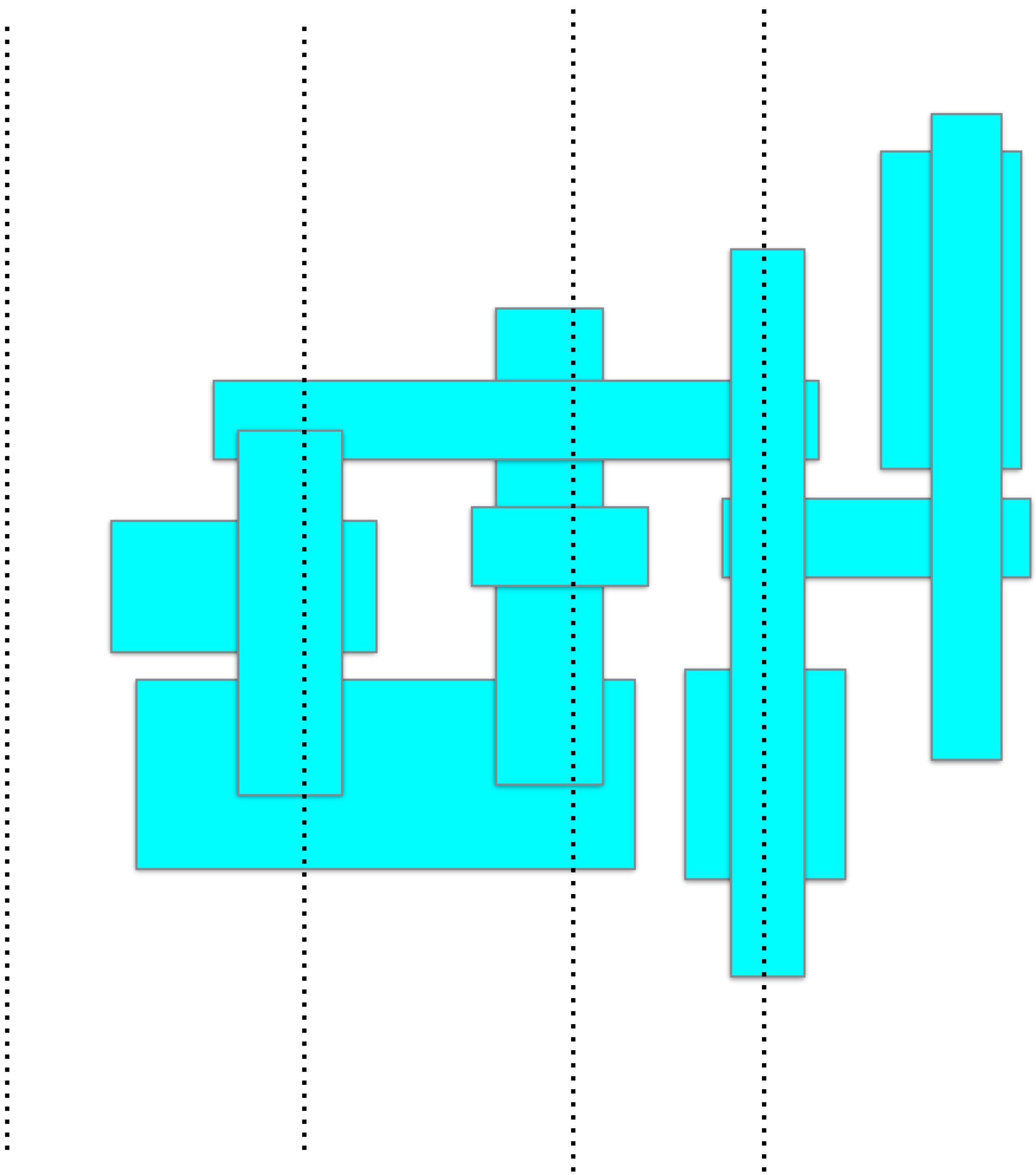
Algorithm

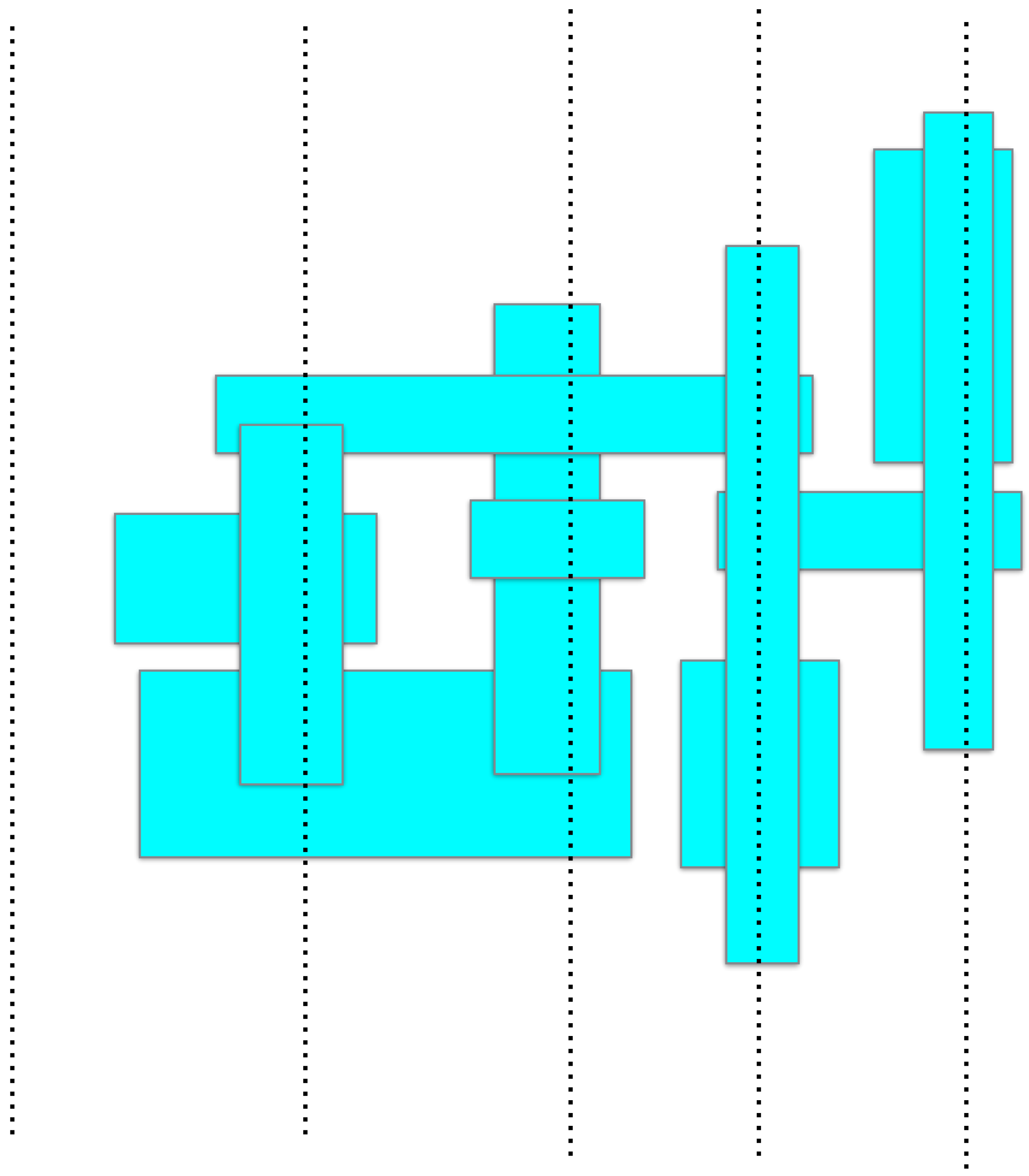


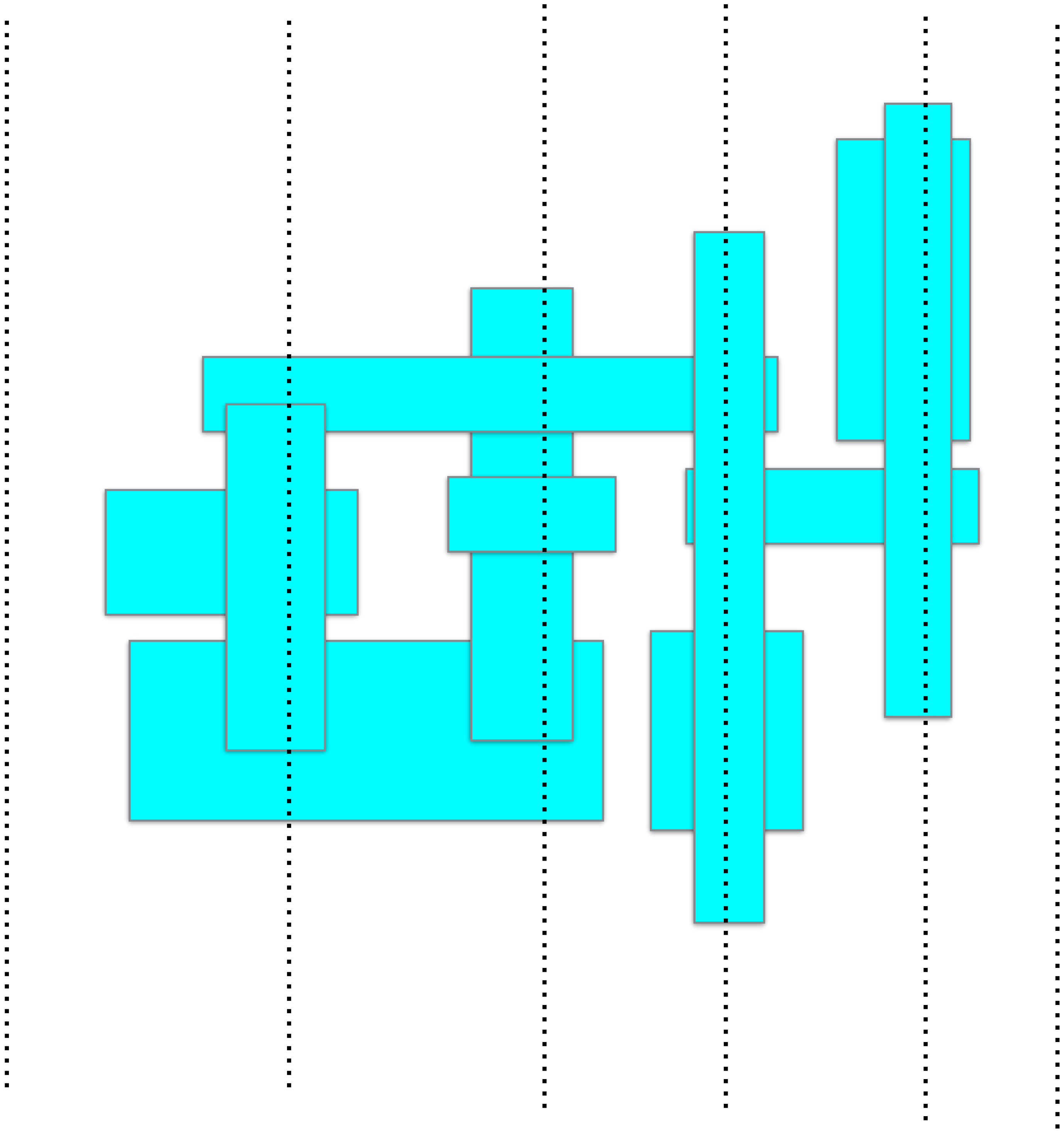


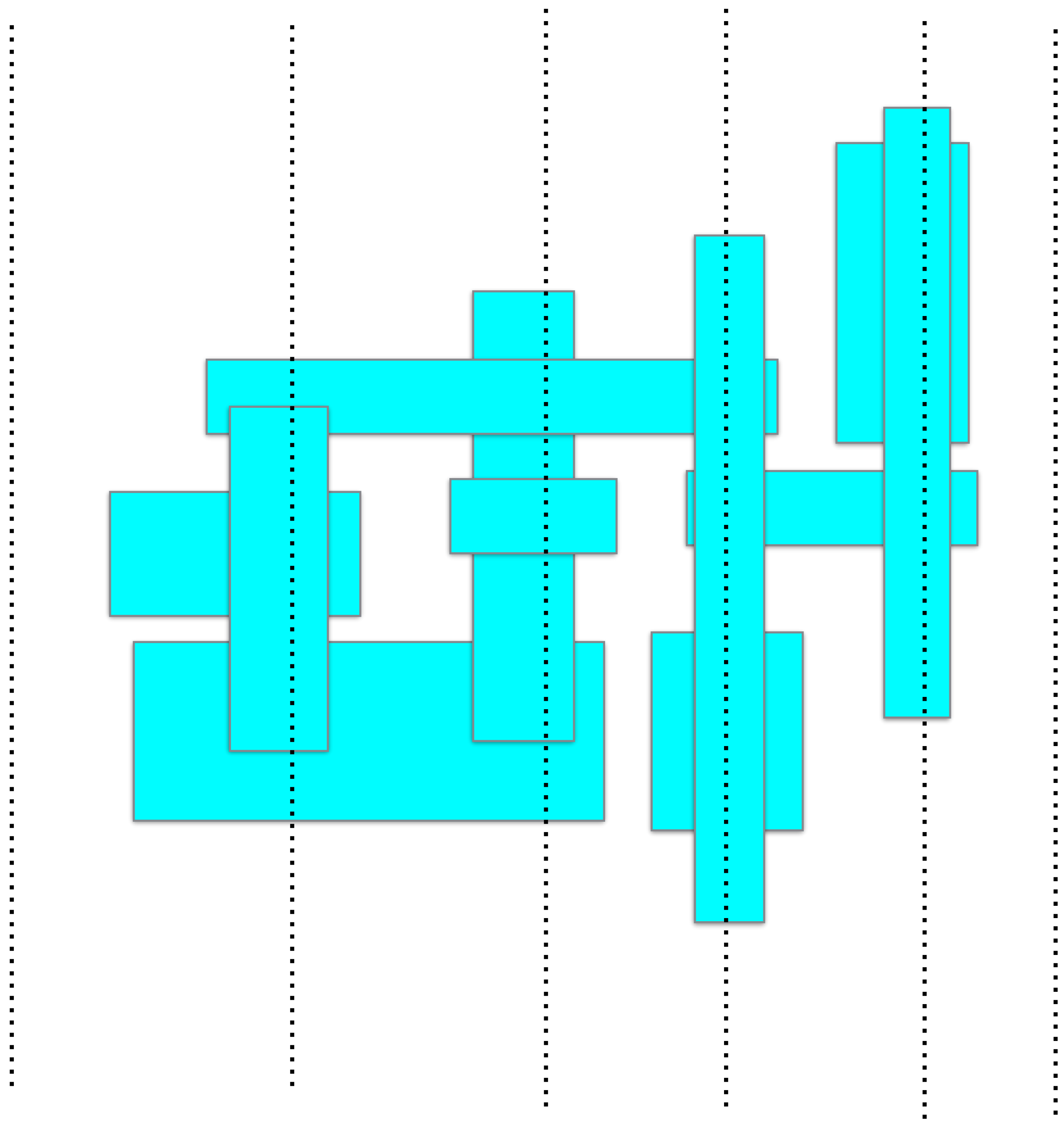






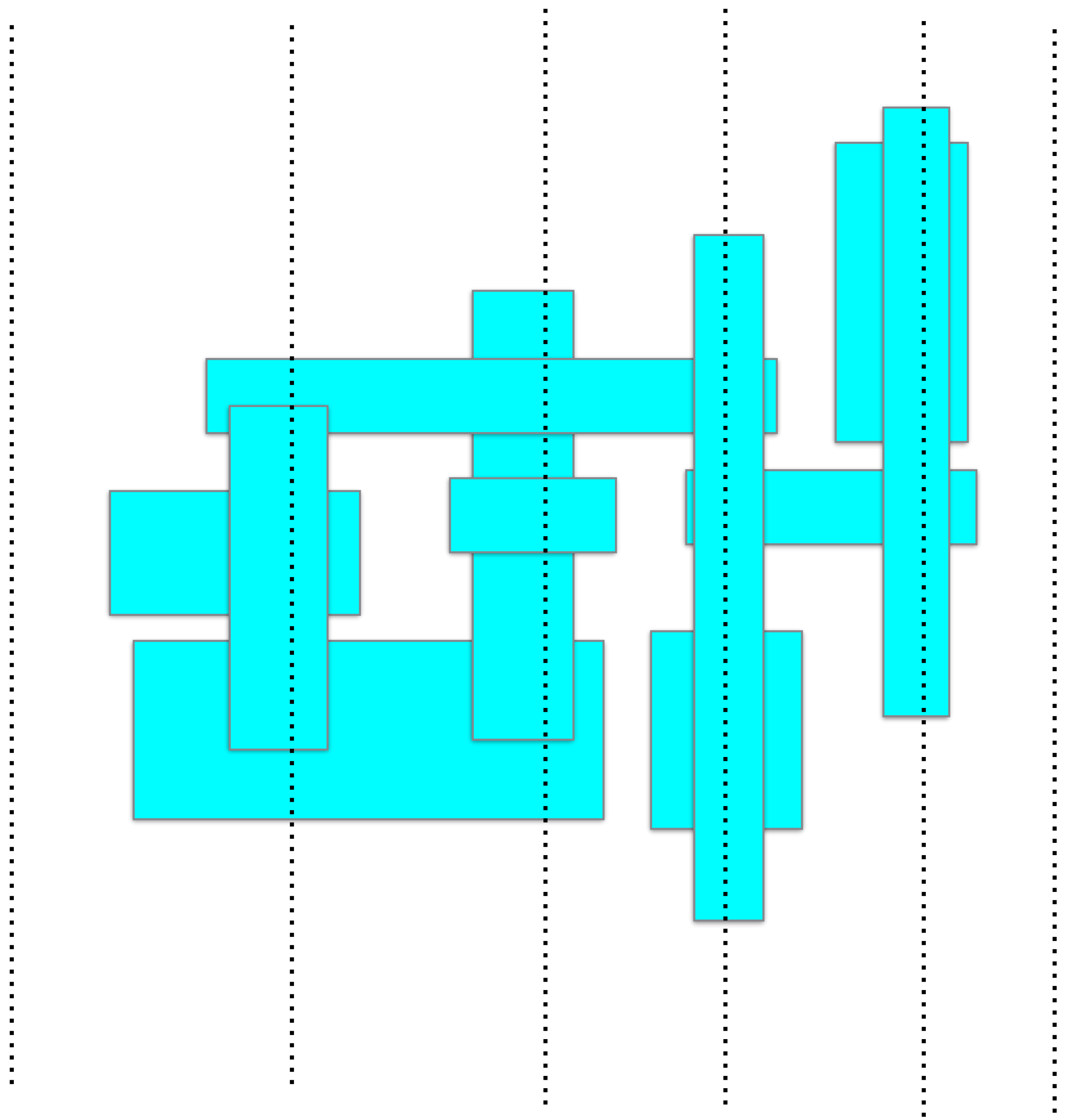






If >k+1 lines,

then we get a solution of size k



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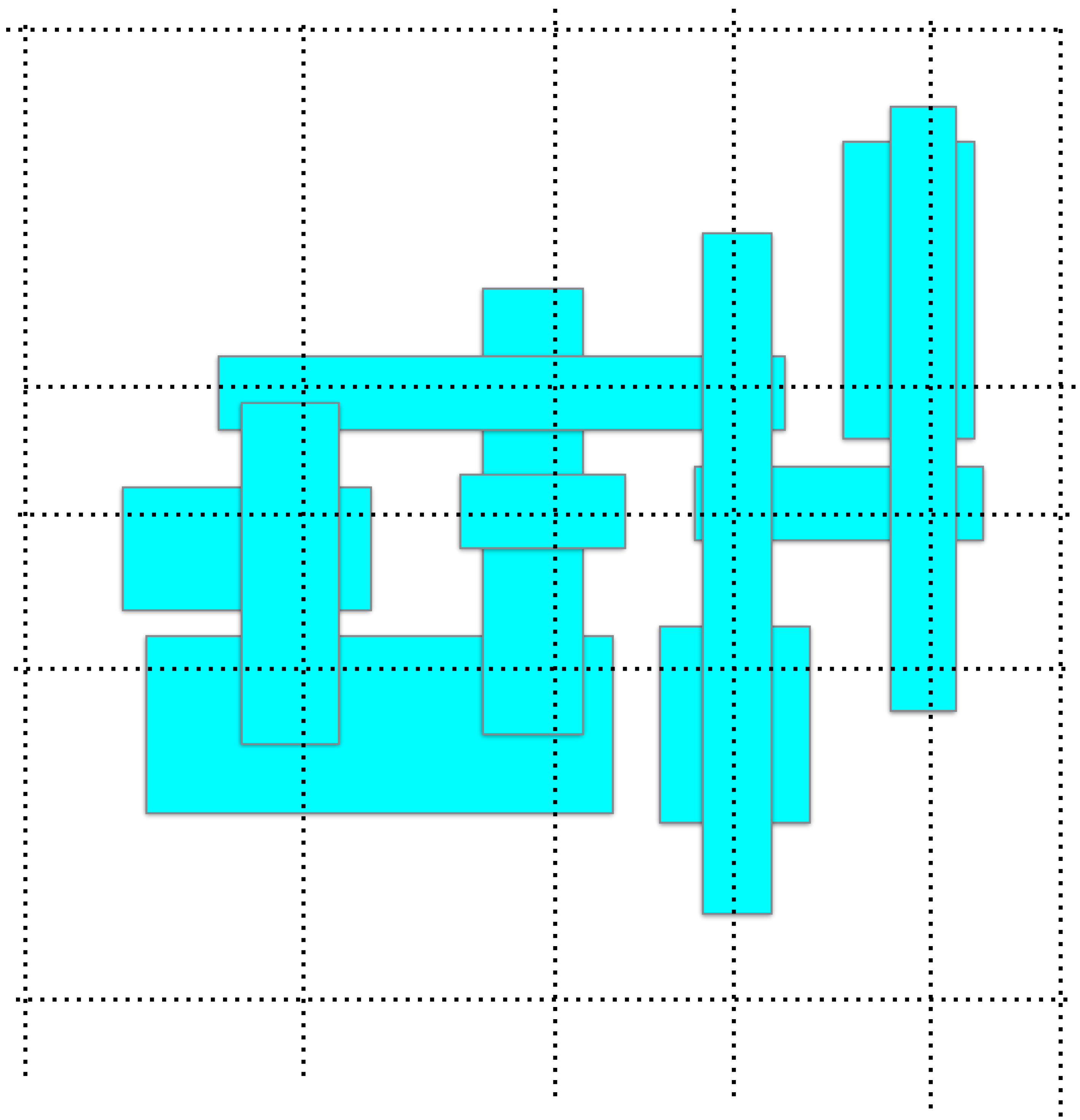
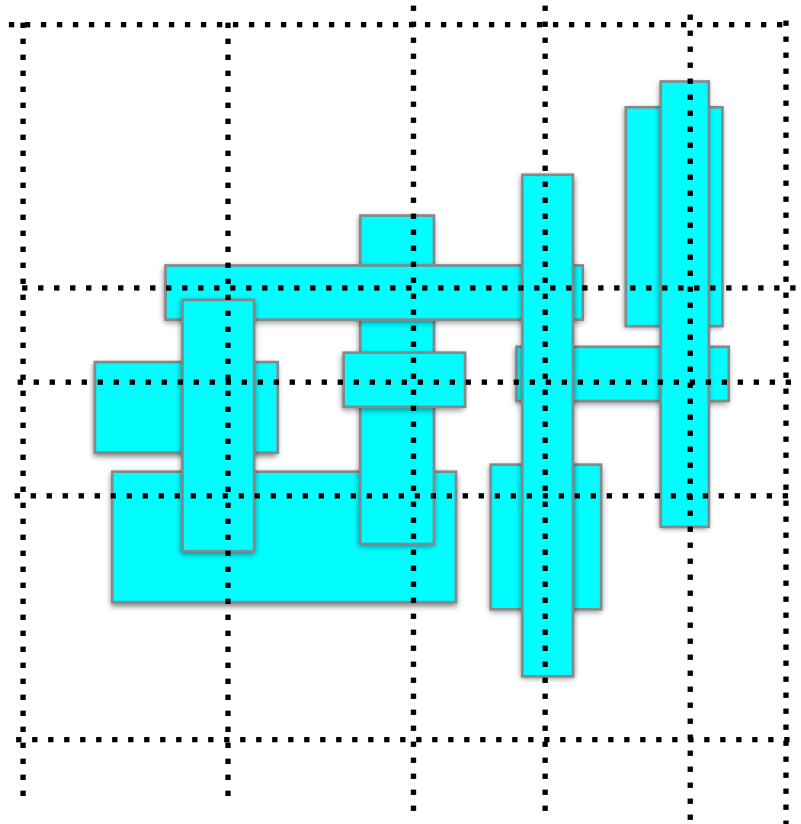


If >k+1 horizontal lines, then we get a solution of size k

Partition Lemma

Let R be a solution of size k.
Then, there exits $S \subseteq R$ s.t.

- $|S| \geq (1-\varepsilon)|R|$



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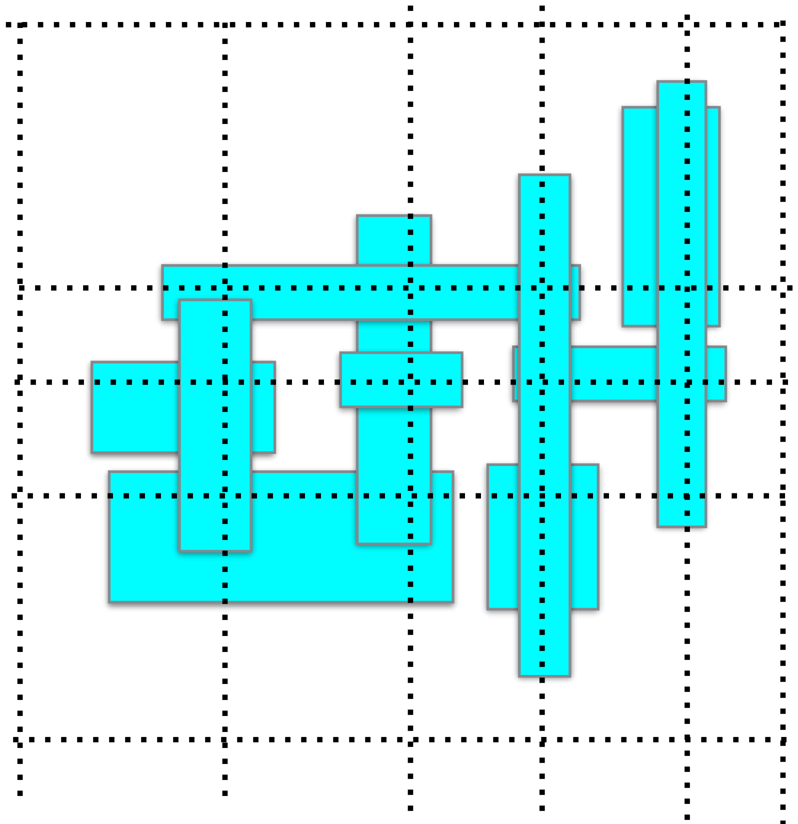

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  ✦ $|S_i| = O(1/\varepsilon^2)$

  ✦ Any cell g in the grid intersects rectangles of S from a block only

Partition Lemma

Let C_i be the set of cells that intersects rectangles in S_i.
Guess C_i for all i. They are disjoint. Find disjoint rectangles Q_i fully contained in C_i, of size at least $|S_i|$.

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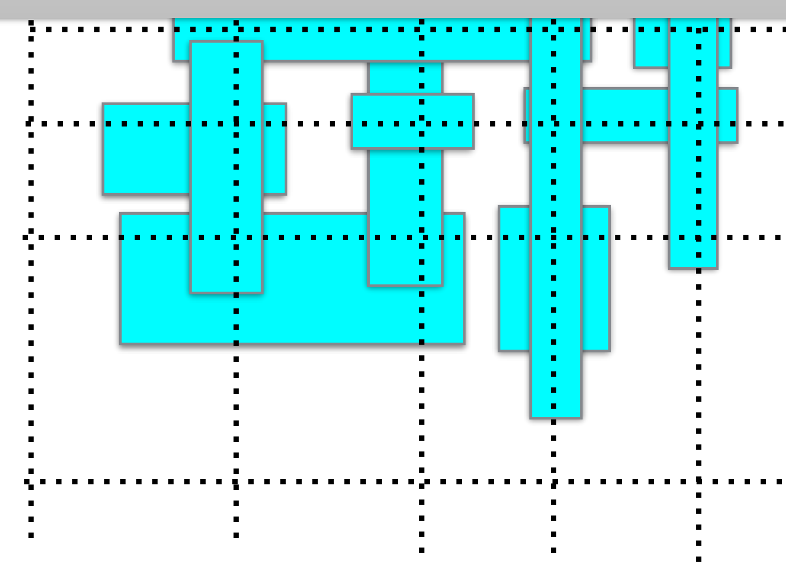

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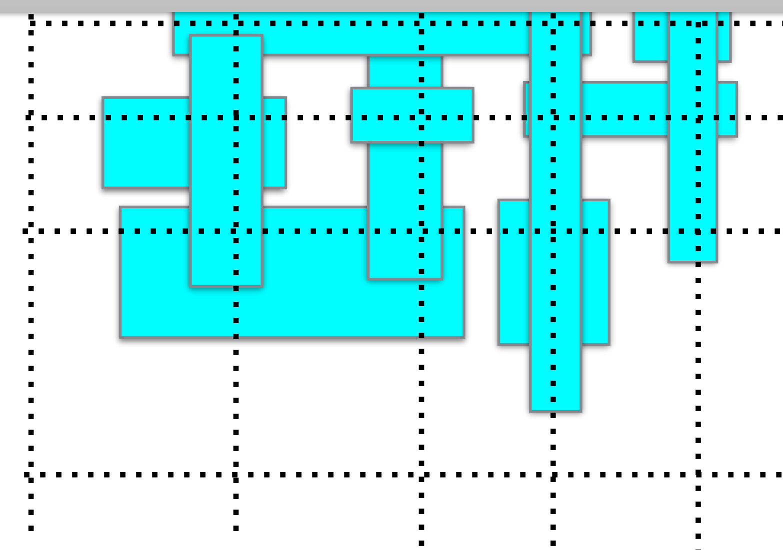

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Running time

★ **Naive:** $2^{O(k^3)} n^{O(1/\varepsilon^2)}$

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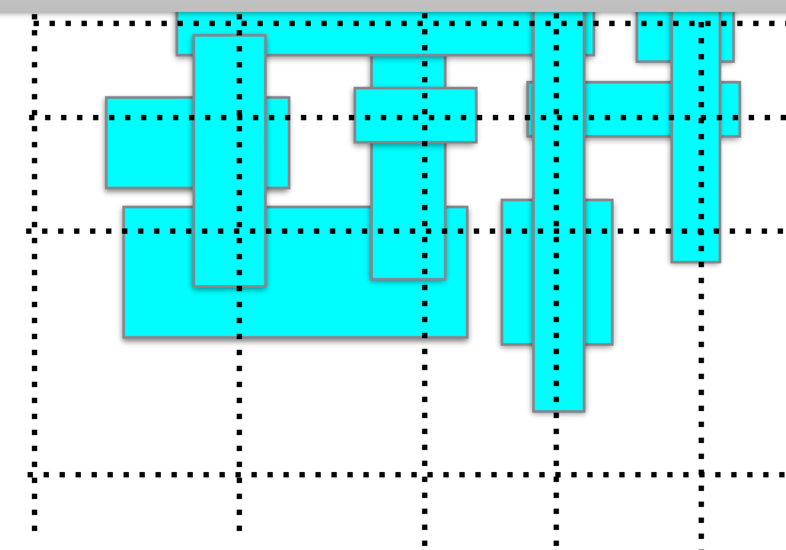

Running time

- ★ **Naive:** $2^{O(k^3)}\, n^{O(1/\varepsilon^2)}$
- ★ **Other:** $k^{O(k/\varepsilon^2)}\, n^{O(1/\varepsilon^2)}$

Proof: Partition Lemma

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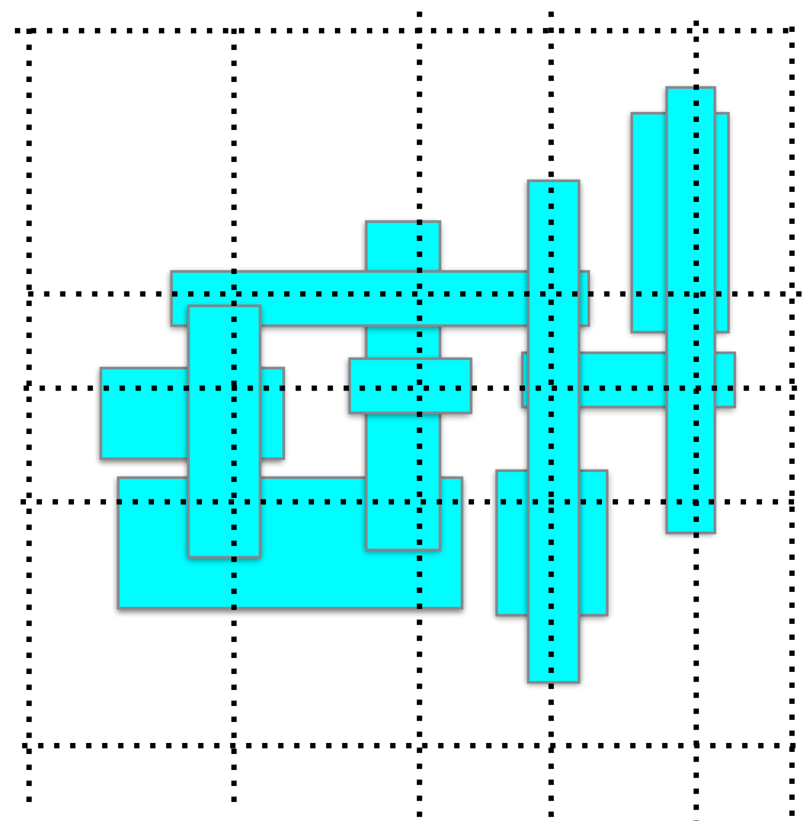

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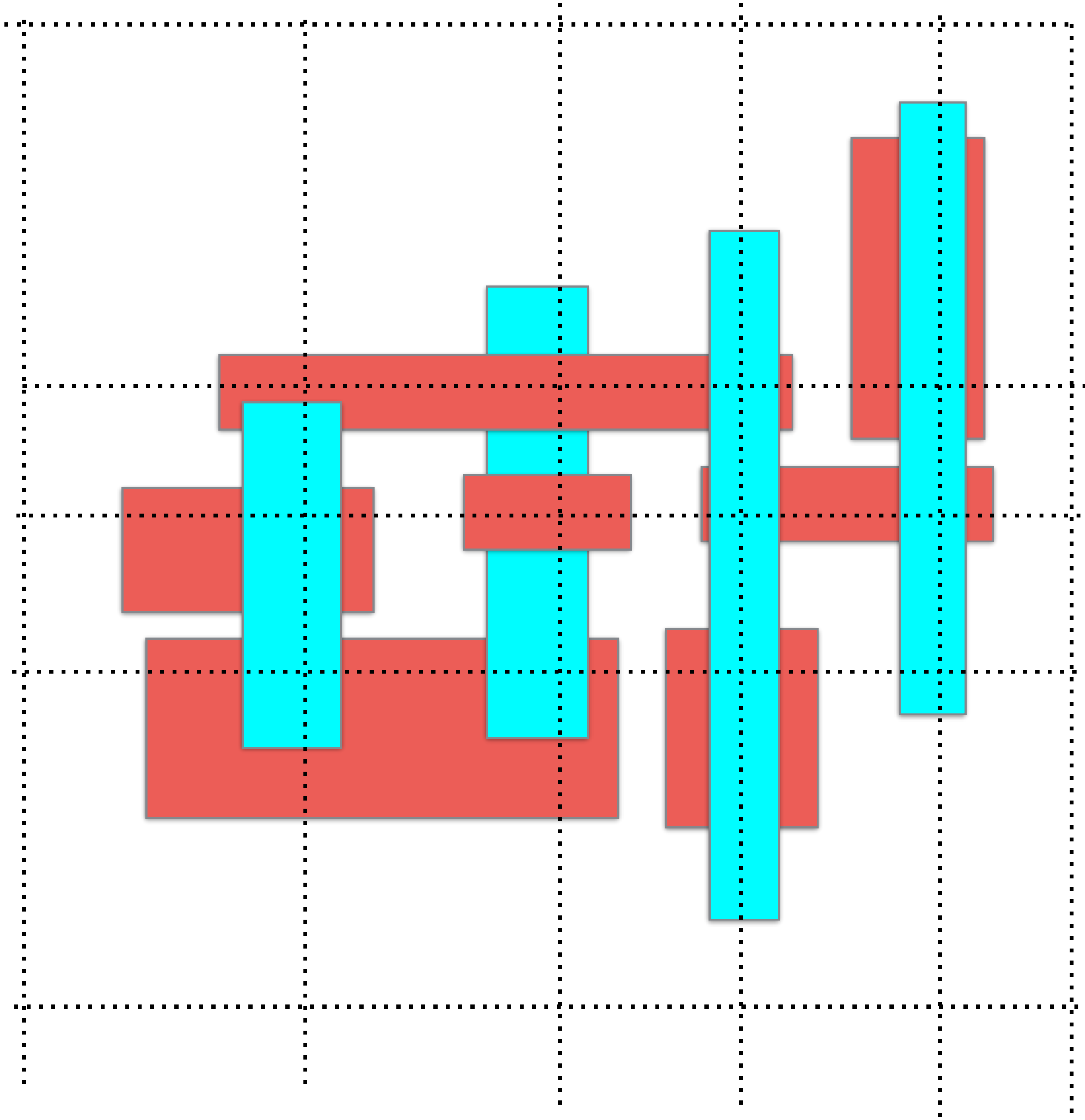

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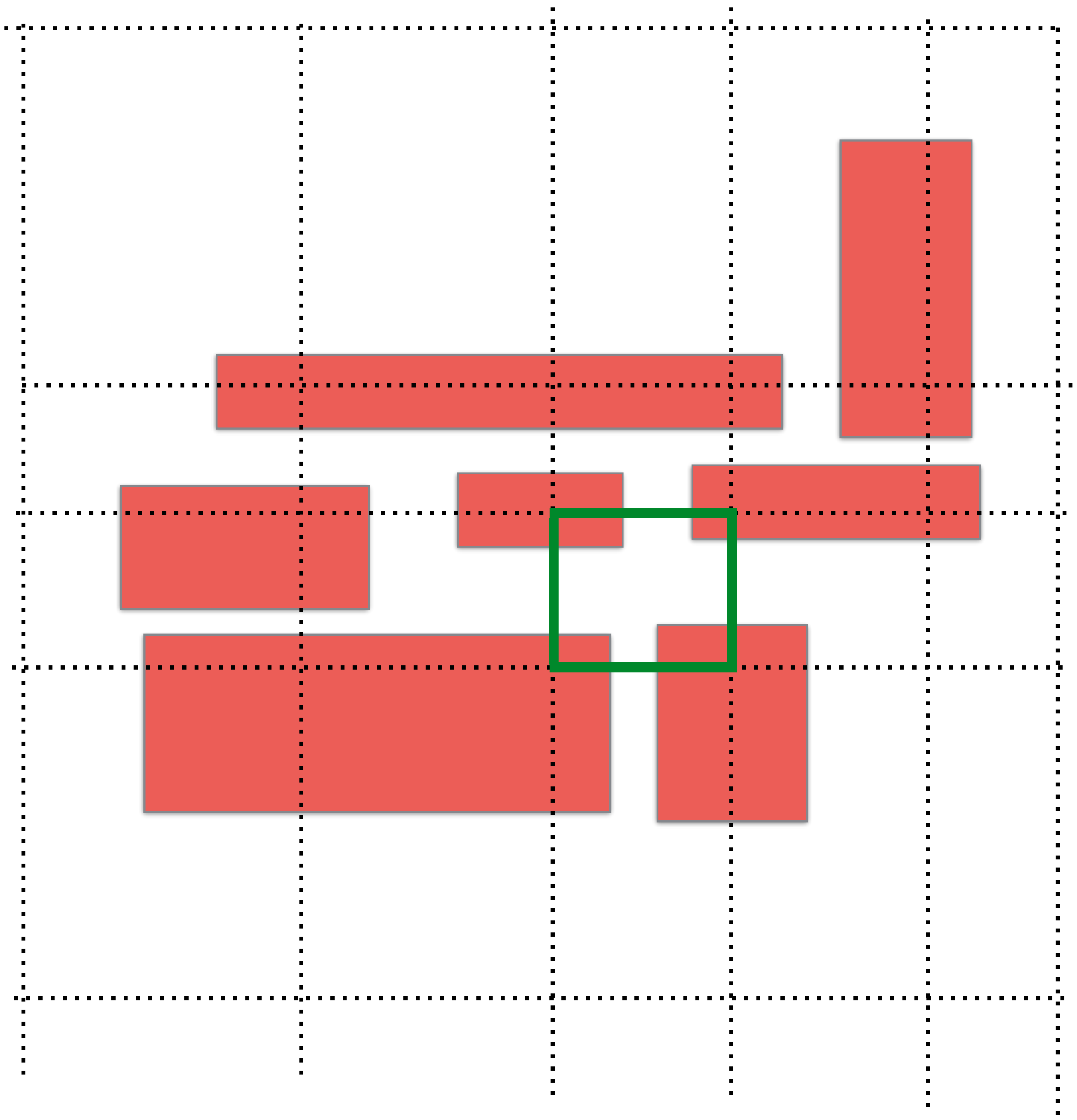

Proof: Partition Lemma

An r-division: An r–division of G is a decomposition into

- O(n/r) edge-disjoint pieces,
- each with ≤ r vertices and
- O(√r) boundary vertices (i.e., vertices with edges in at least two pieces). [That is, total no. of boundary vertices is O(n/√r)]

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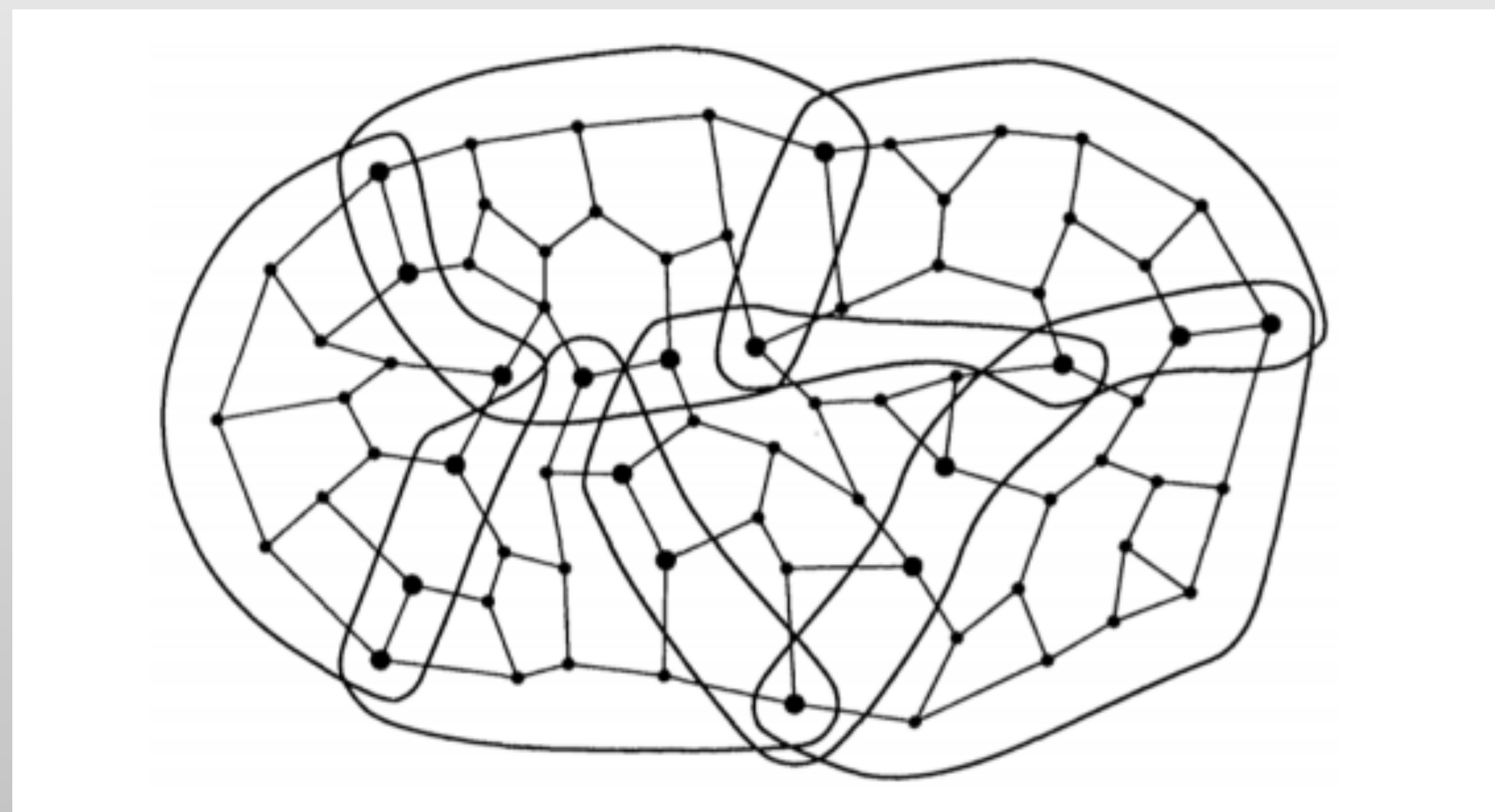

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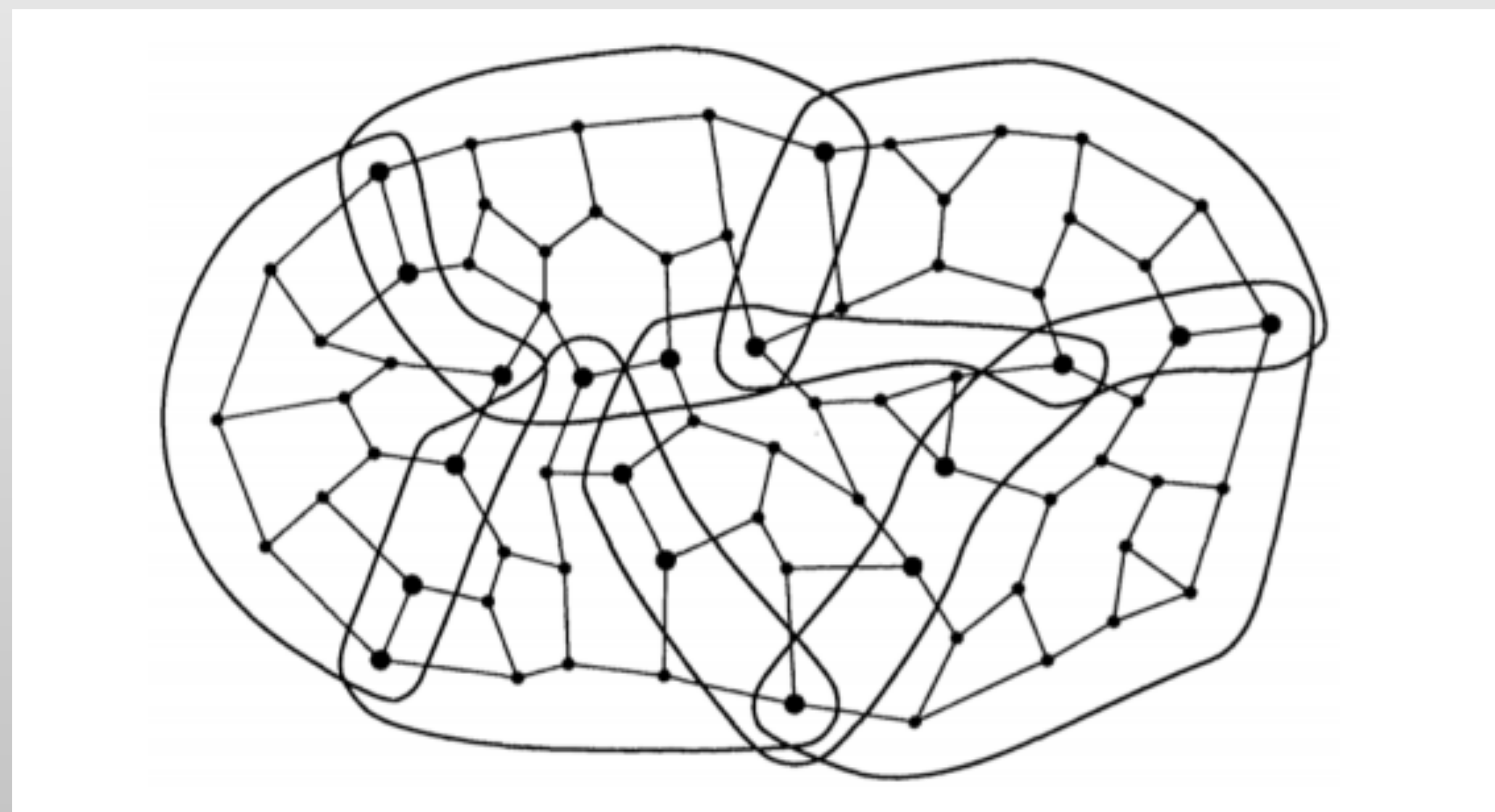

Planar graph admits an r–division

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For any n-vertex planar graph G and an integer r, there exists (n/√r) vertices B such that the number of vertices in each connected component of G-B is at most O(r).

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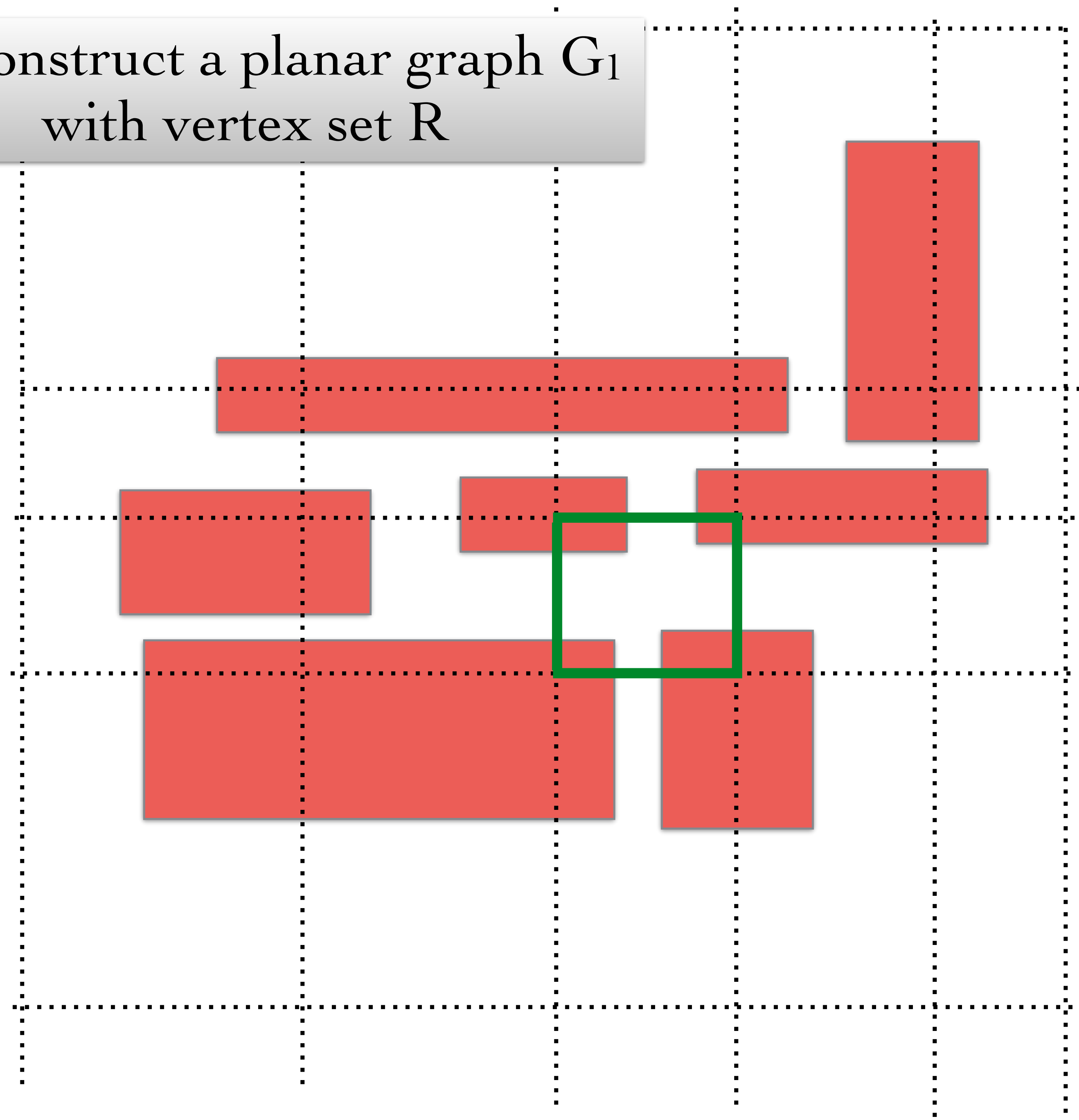
For any n-vertex planar graph G and $0<\delta<1$, there exist δn vertices B such that the number of vertices in each connected component of G-B is at most $O(1/\delta^2)$.

Planar graph admits an r–division

Proof: Partition Lemma

We construct a planar graph G_1 with vertex set R



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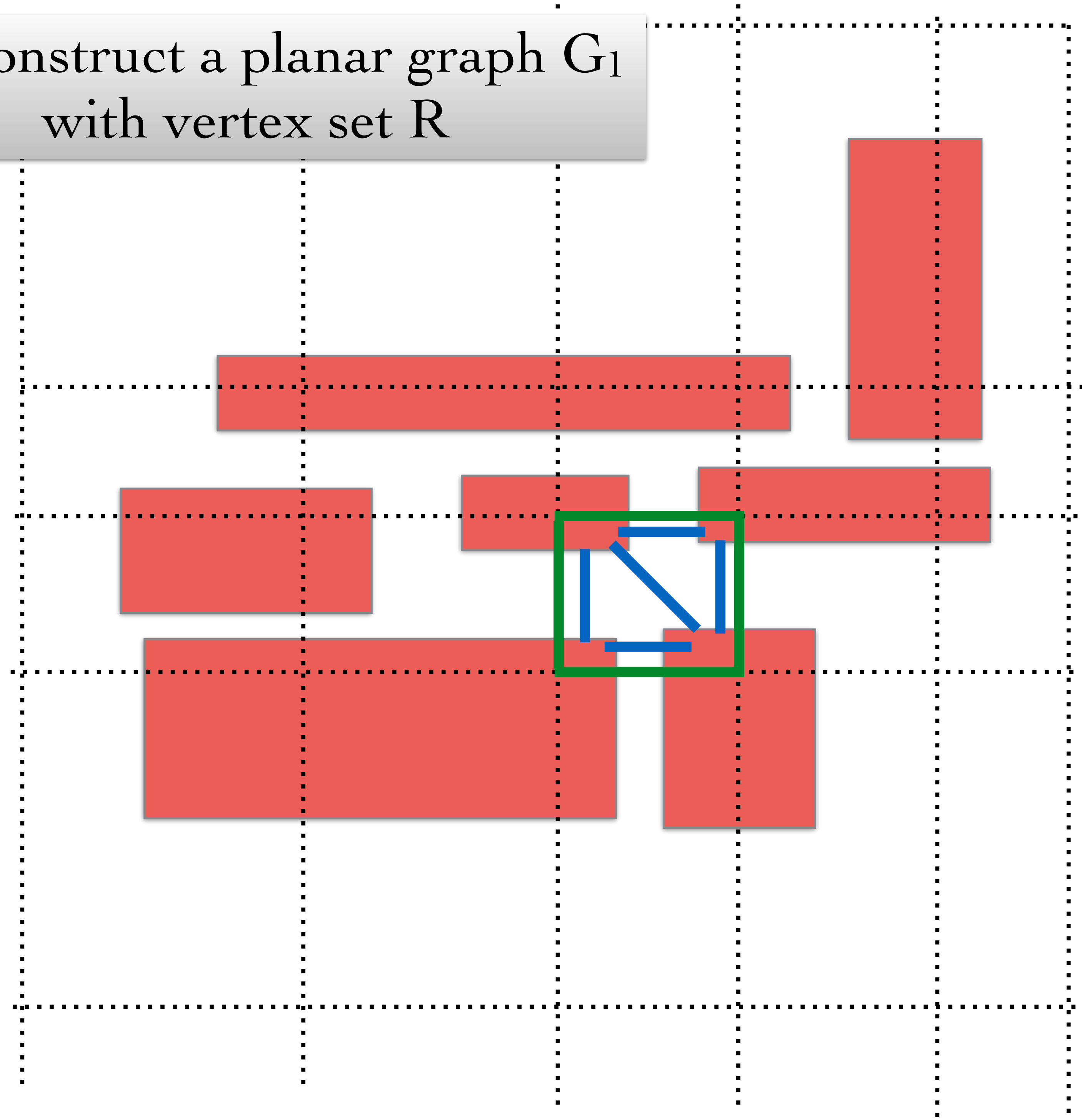

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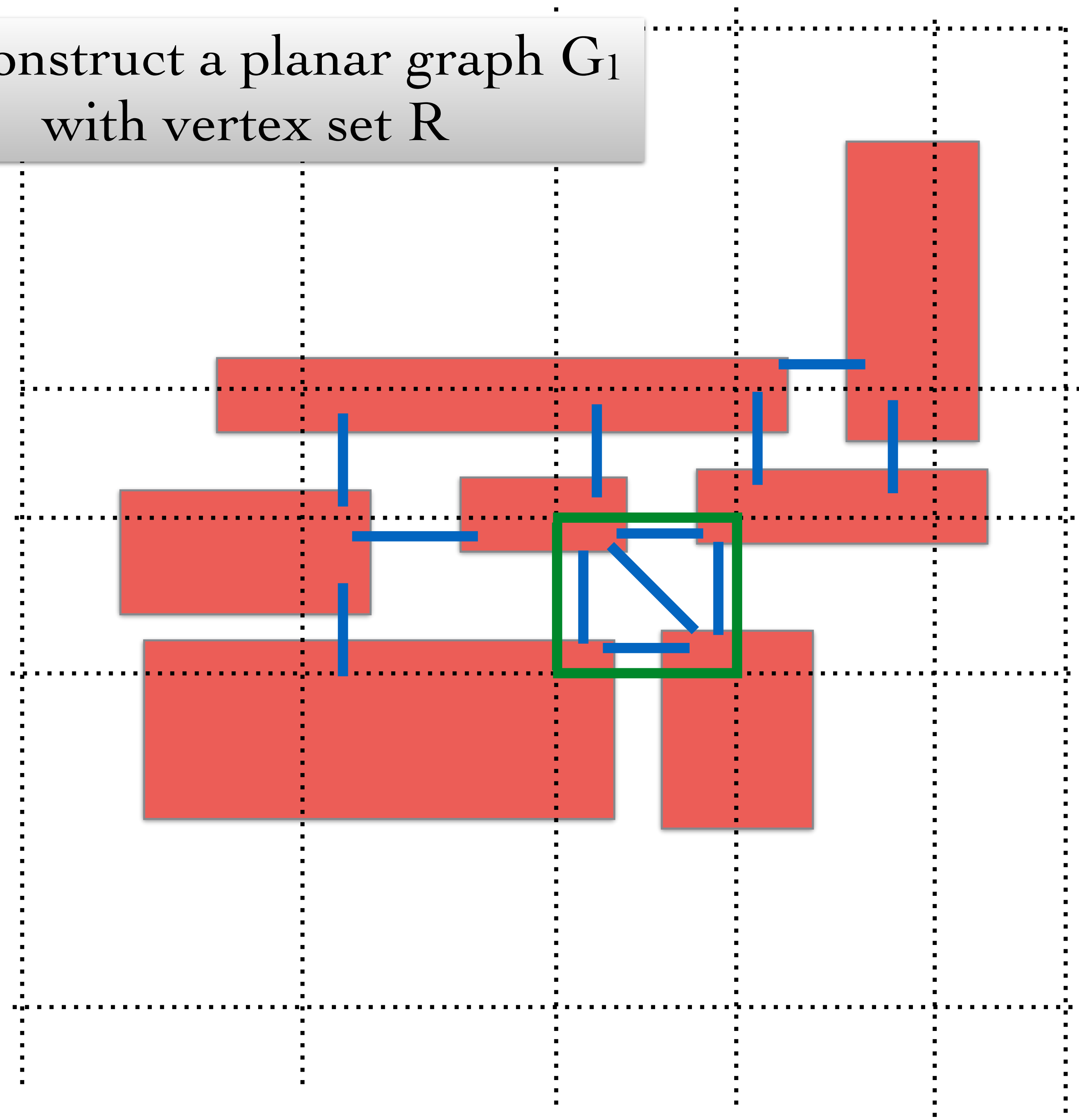
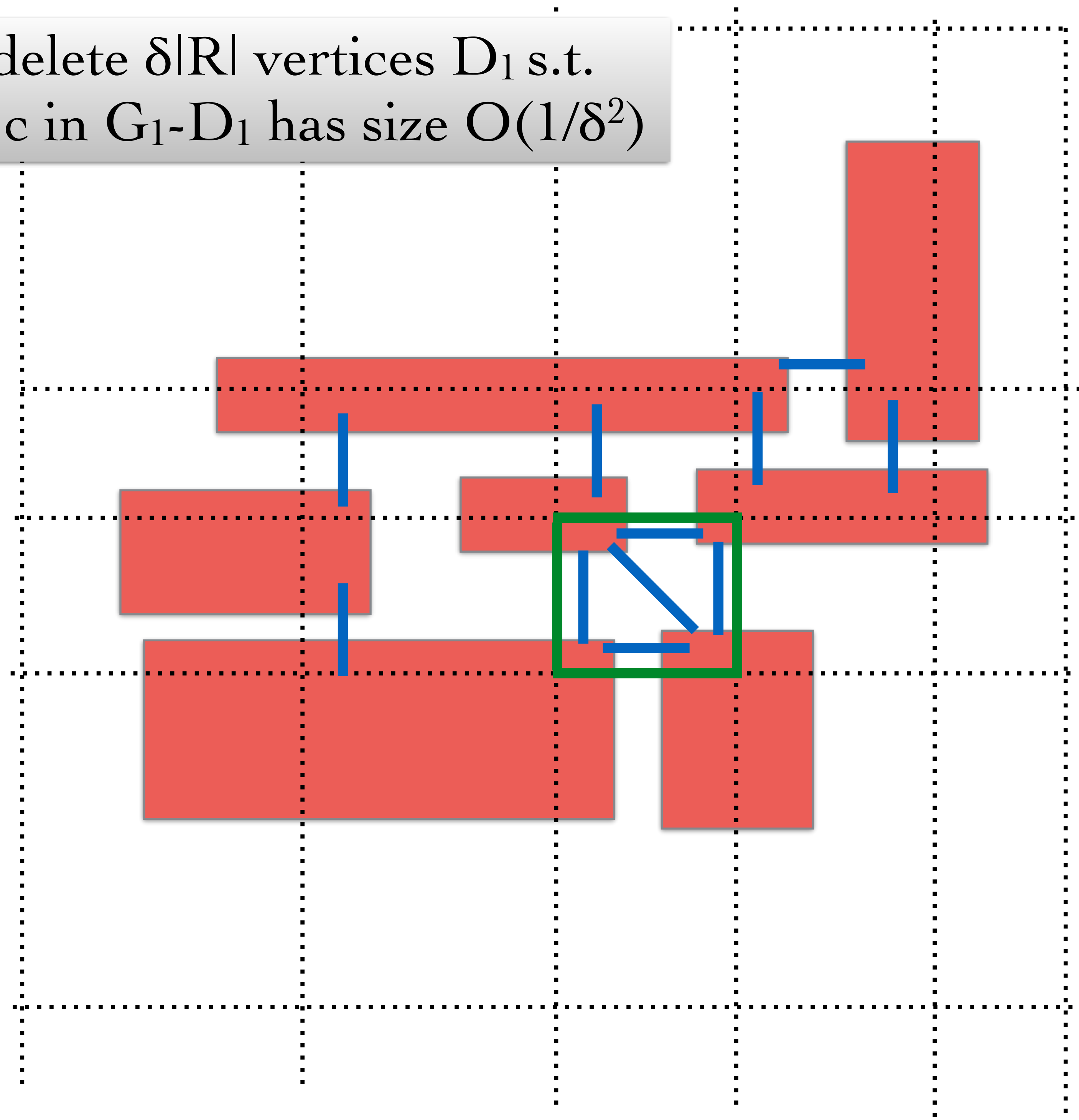


Proof: Partition Lemma

We delete $\delta|R|$ vertices D_1 s.t. each c.c in G_1-D_1 has size $O(1/\delta^2)$



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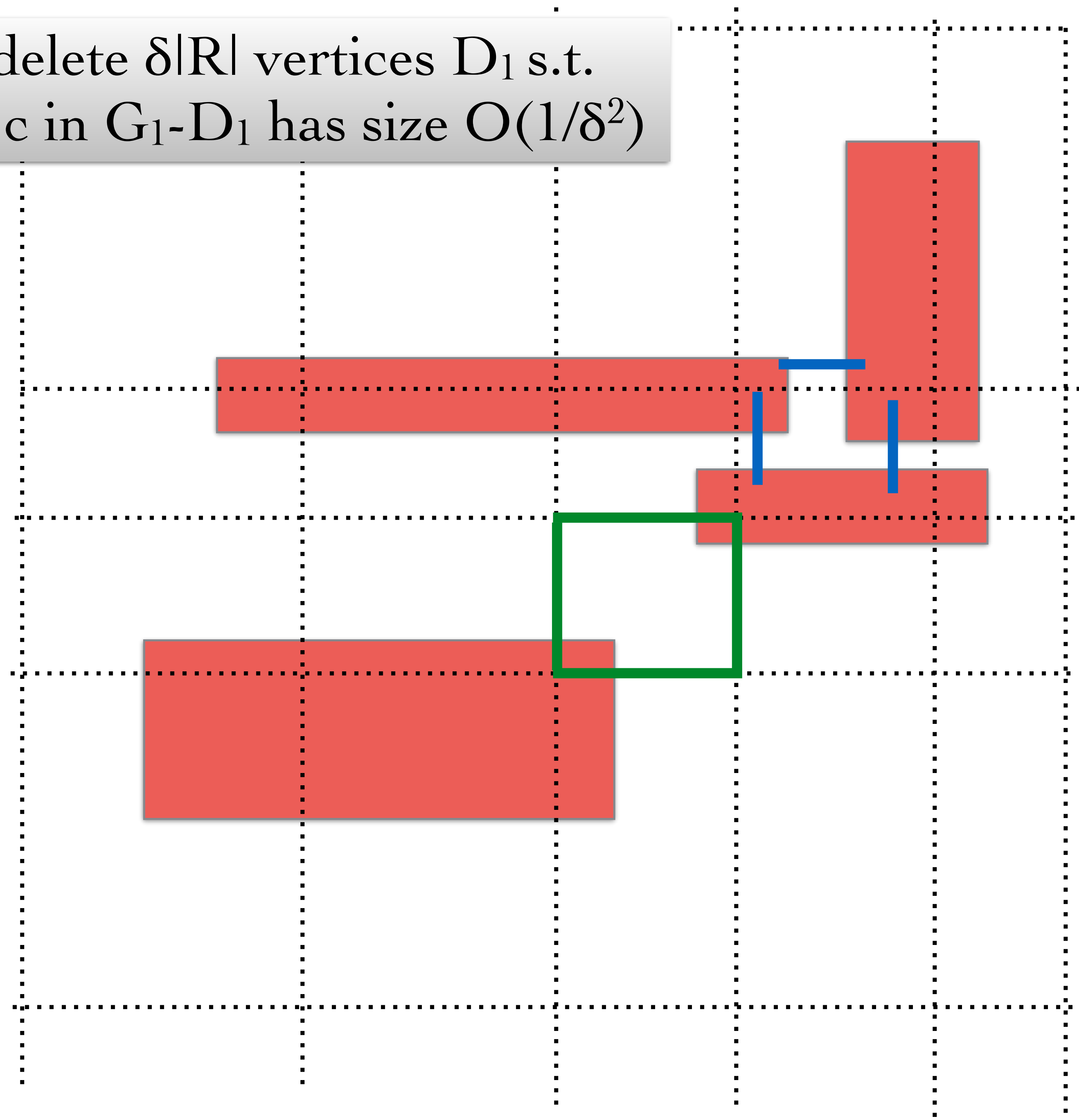

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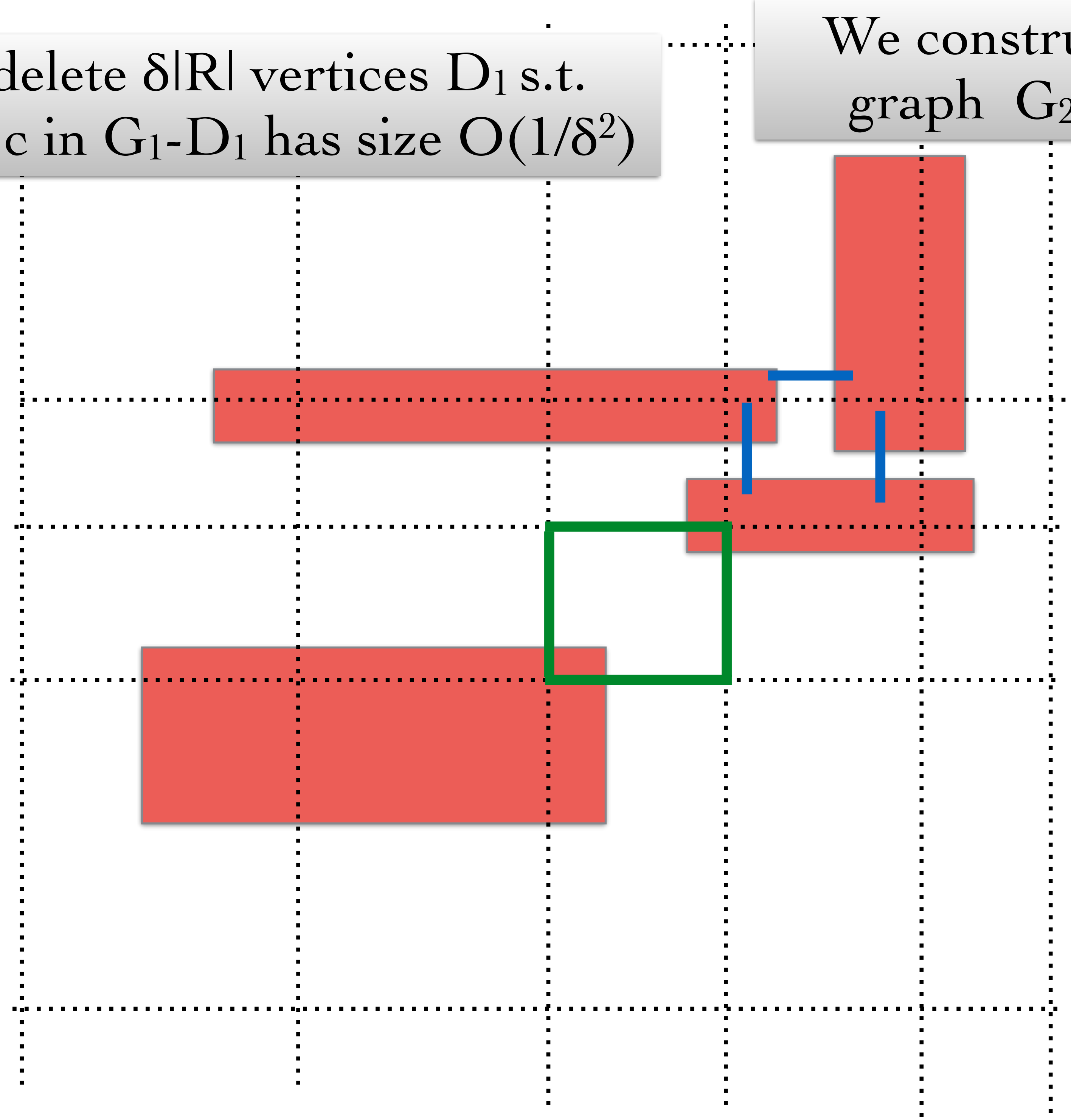

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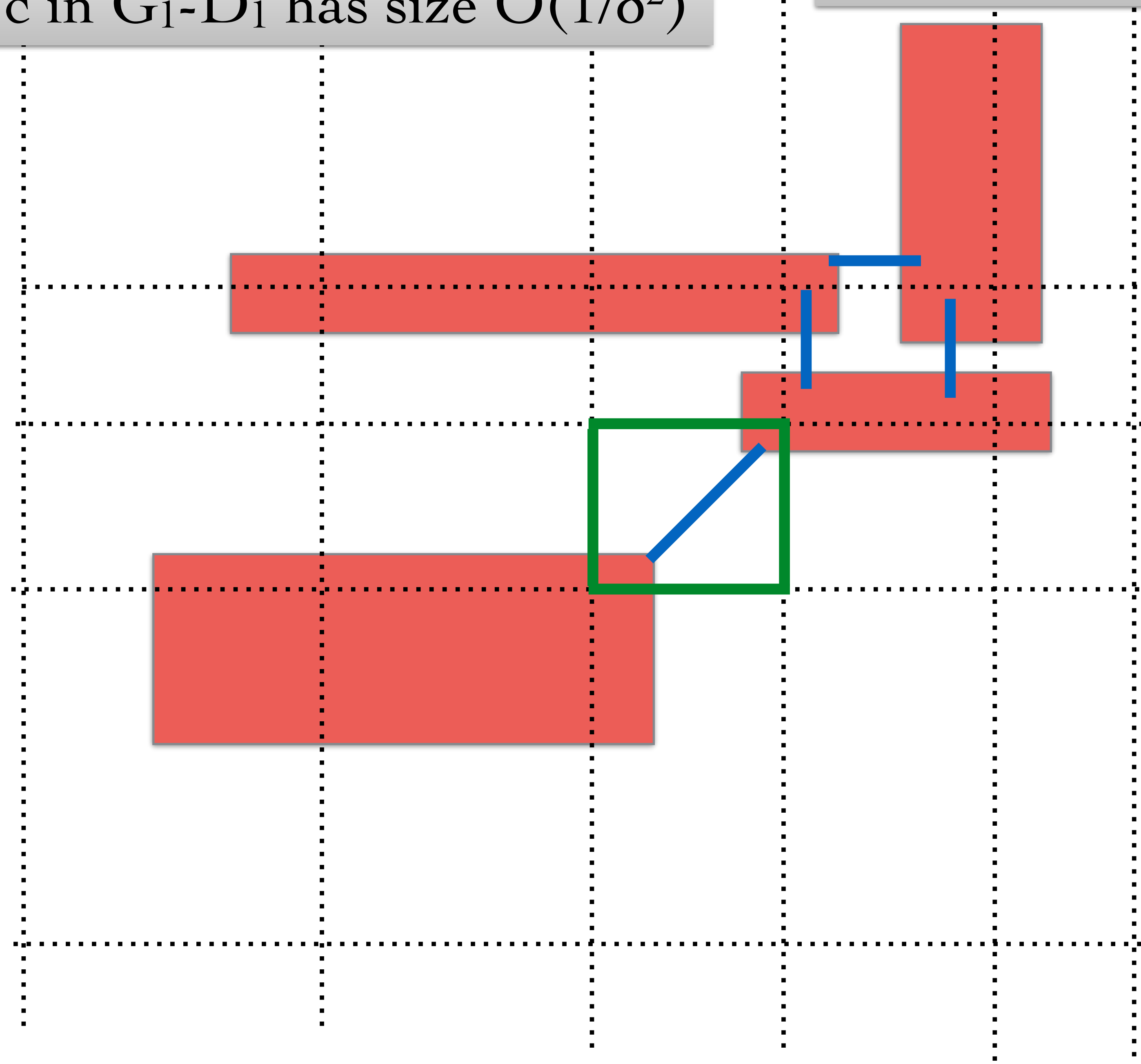

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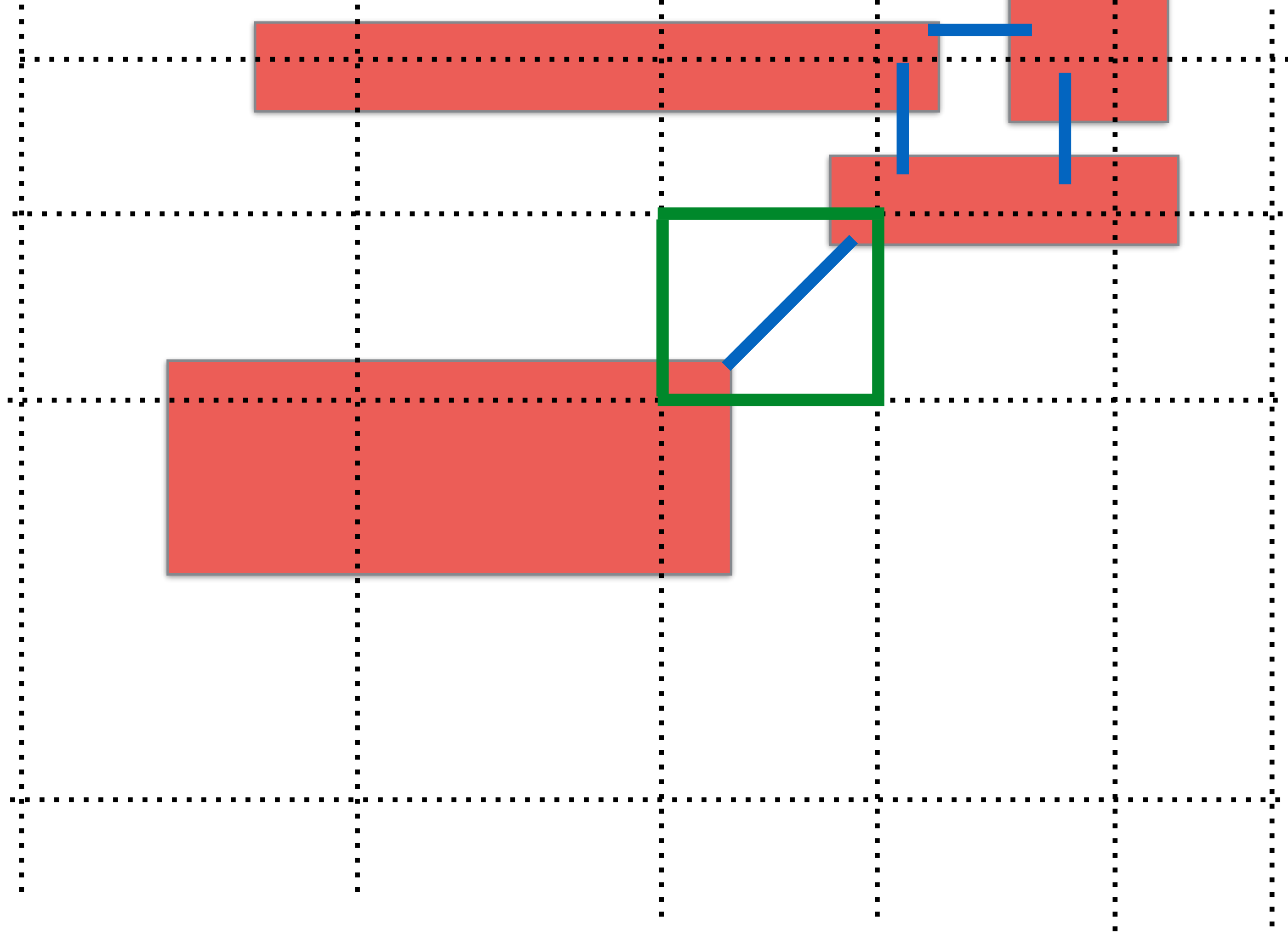

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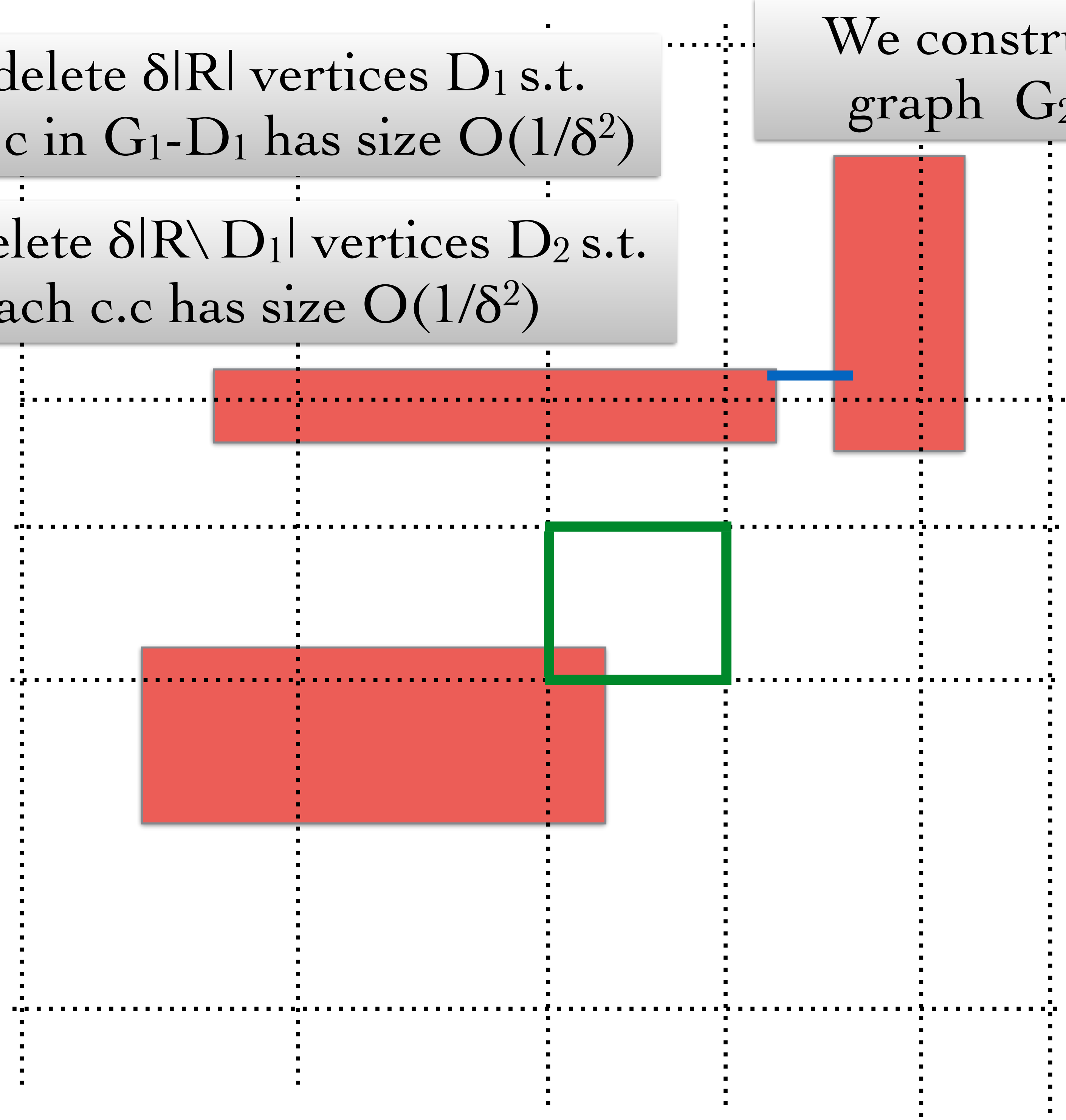

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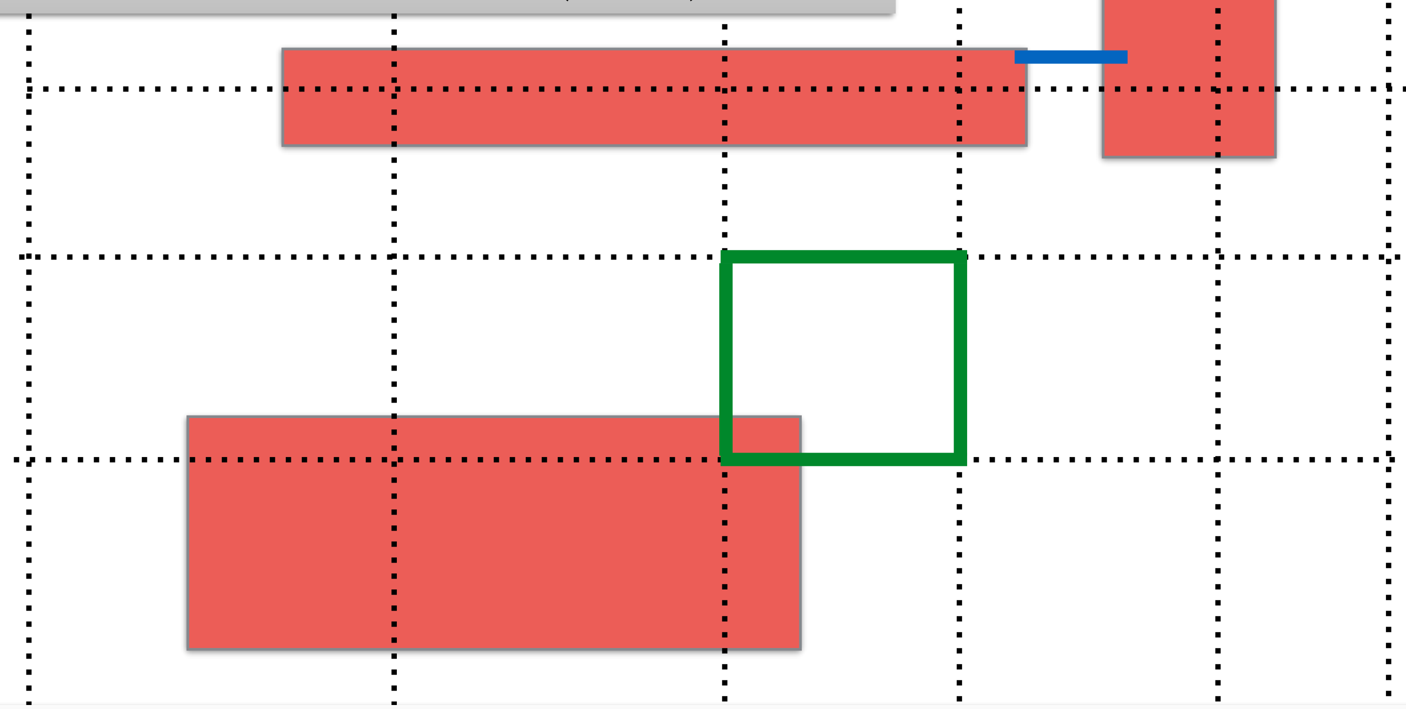

$|R \setminus D_1 \setminus D_2| \geq (1-2\delta)|R|$

The connected components of G_1-D_1-D_2 gives the required partition

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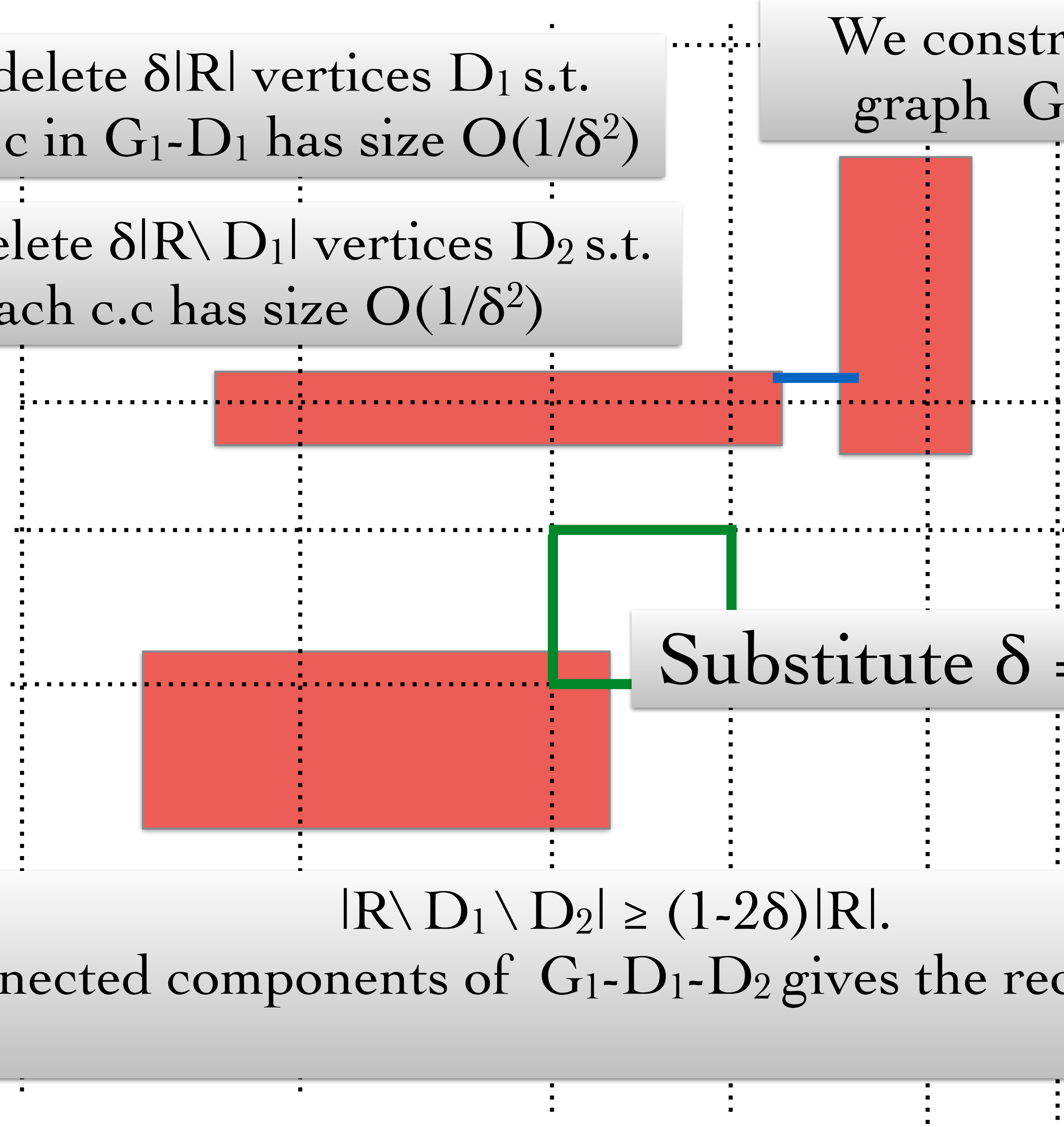

Substitute $\delta = \varepsilon/2$

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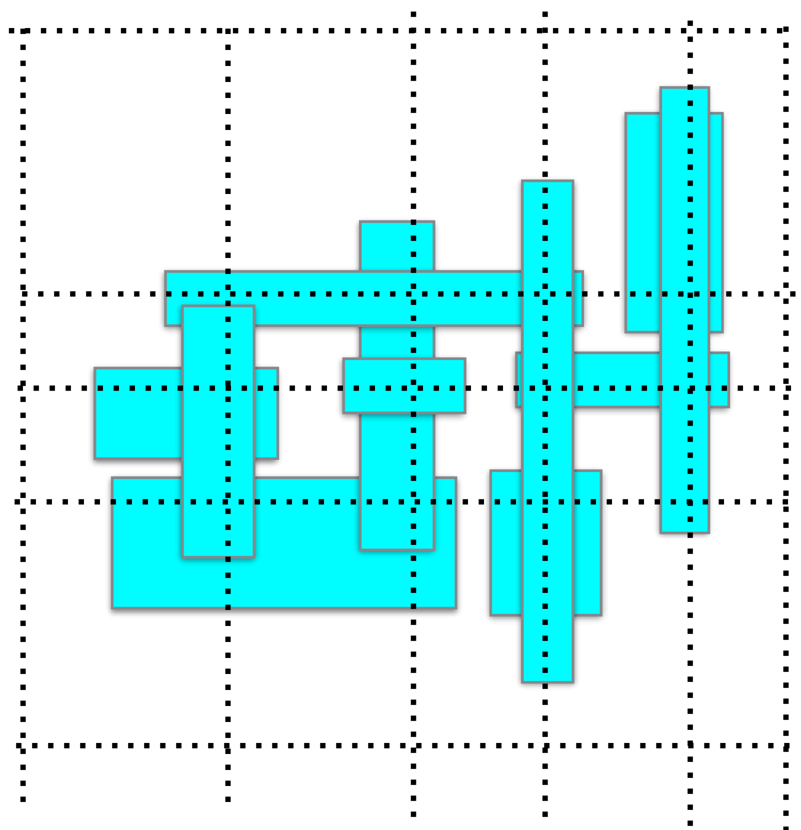

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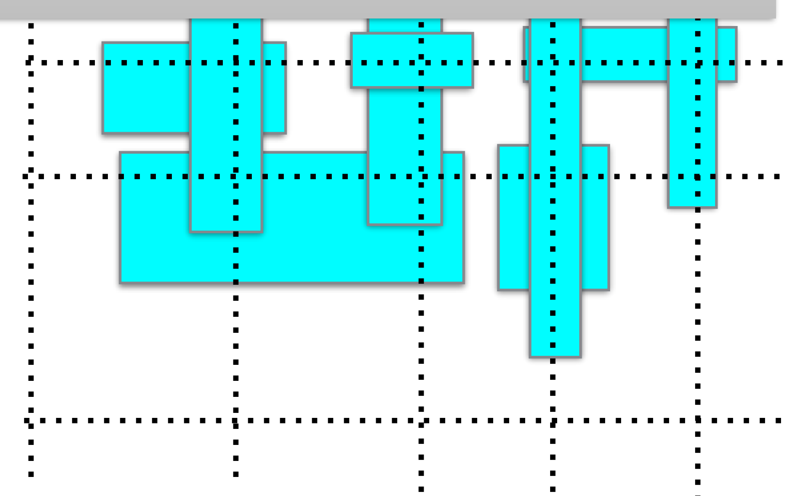

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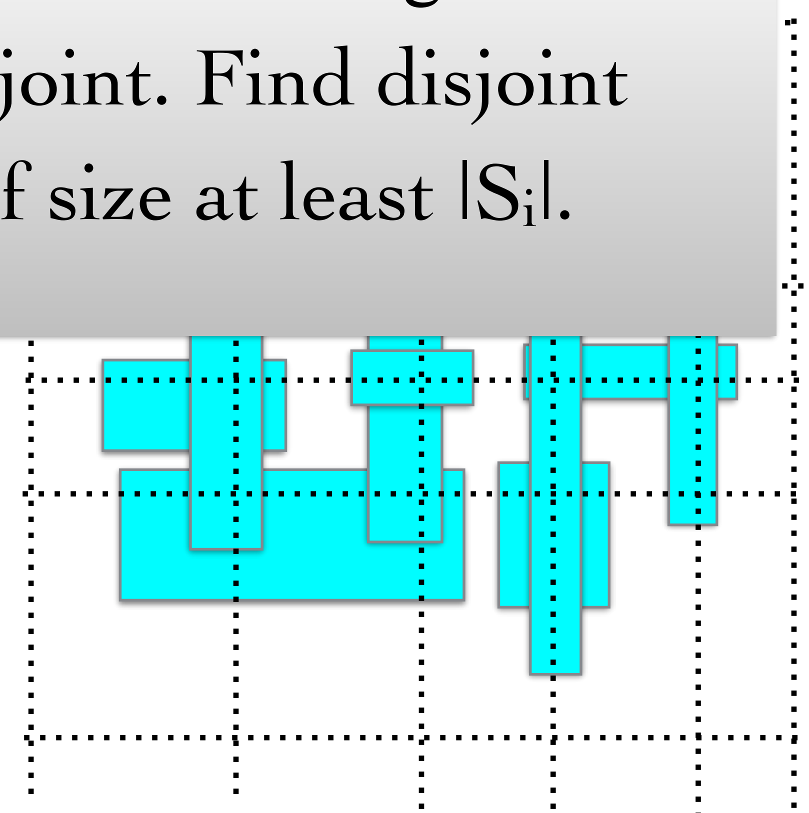

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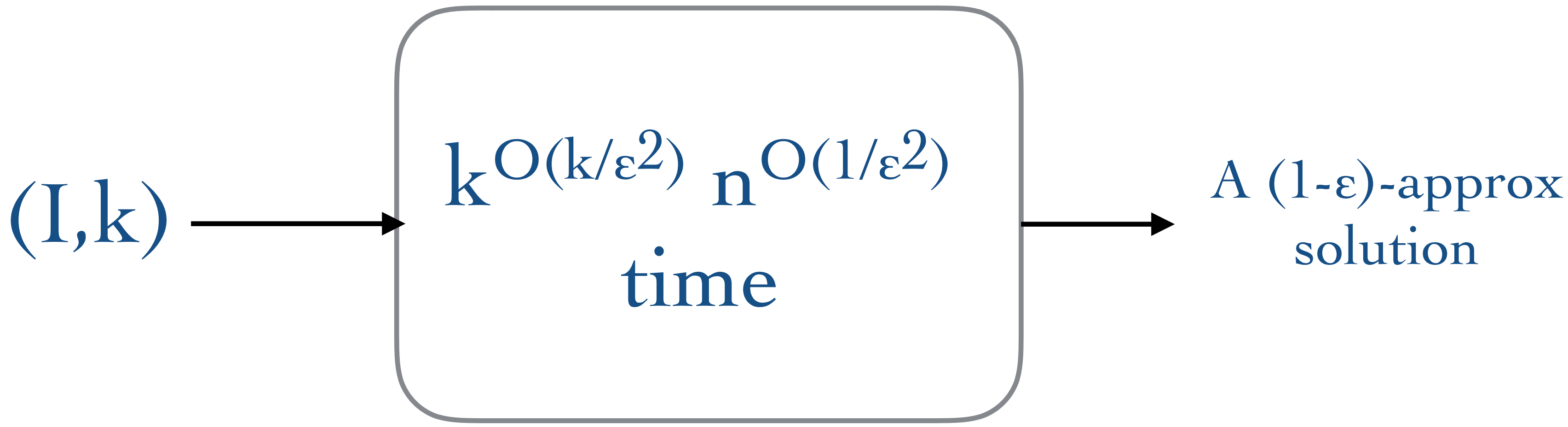
★ **No. choices for C_i is $k^{O(1/\varepsilon^2)}$**

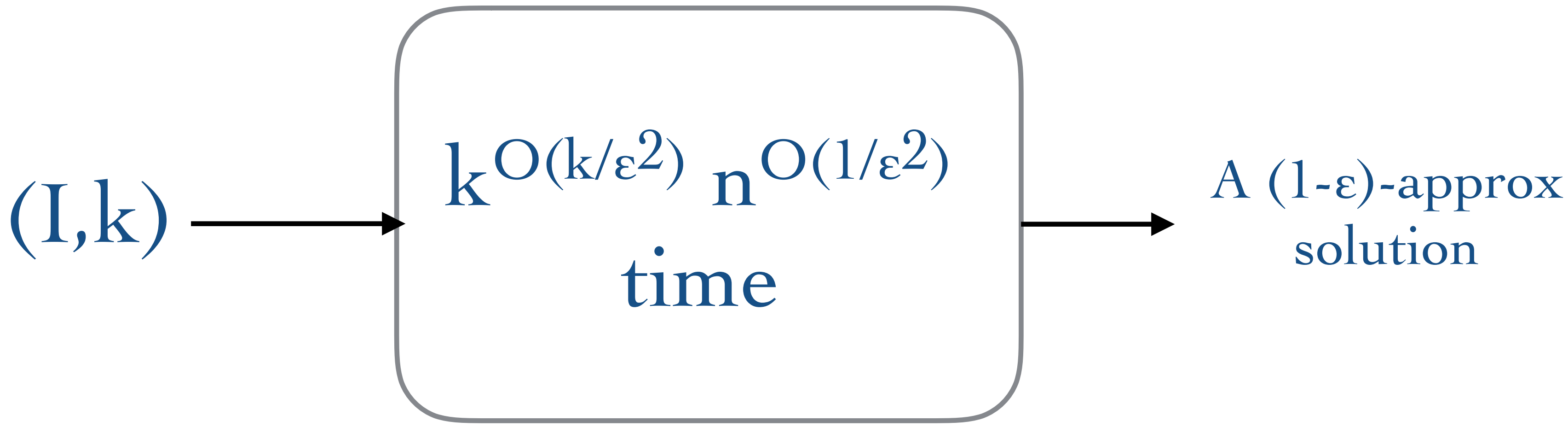
★ **One can get $k^{O(1/\varepsilon^2)}$ $(1/\varepsilon^2)$ rectangles containing a solution of size $\geq(1-\varepsilon)$k**

Summary



Parameterised Approximation Scheme (PAS)

Summary



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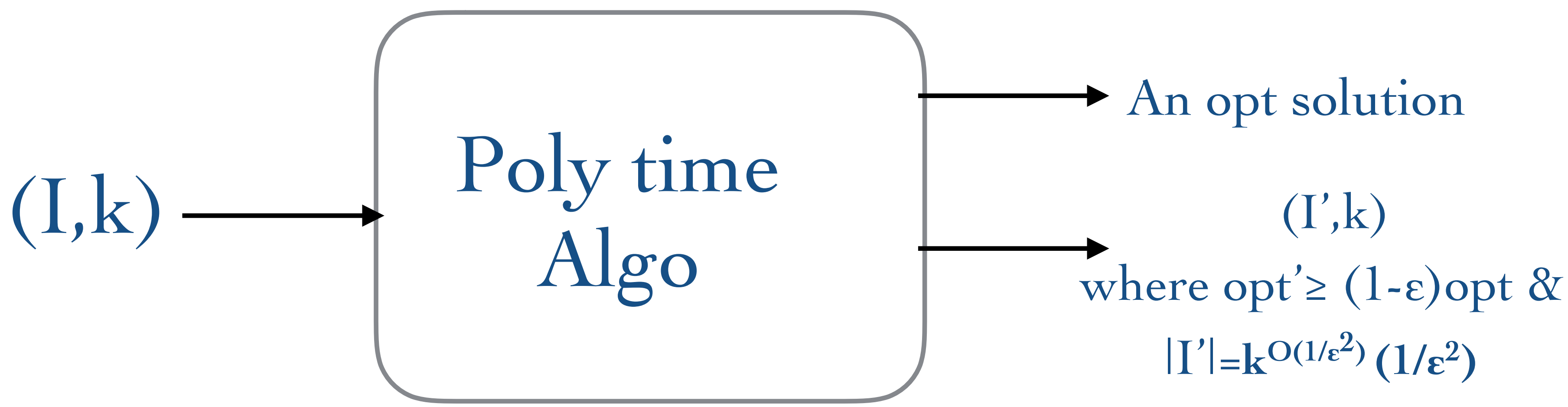

Polynomial Size Approximate Kernelization Scheme (PSAKS)

Open Problem

- Is there PTAS? Or at least a polynomial time constant factor approximation algorithm?

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- Is there PTAS? Or at least a polynomial time constant factor approximation algorithm?
- The answer to the above question is most likely yes, as there is a QPTAS. But we don't have it yet.