Introduction to channel coding

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What is the maximum rate at which we can reliably communicate across a noisy channel?


1 2 3 4 5 6 7 8 9 10 11 12
JL YL JP YP YJ YK JS YS YR JK JR JW
MOBILE
HORN
OFF
MOISTURE WILL


Rcl Clr Snd
Sto Fcn End
Pwr Lock Vol
British
TELECOM


NOKIA
NOKIA


Sony Ericsson
12:37
Mobile networks
New search
Select network
Search mode
Preferred networks
GSM/3G networks
Info
Select
Back

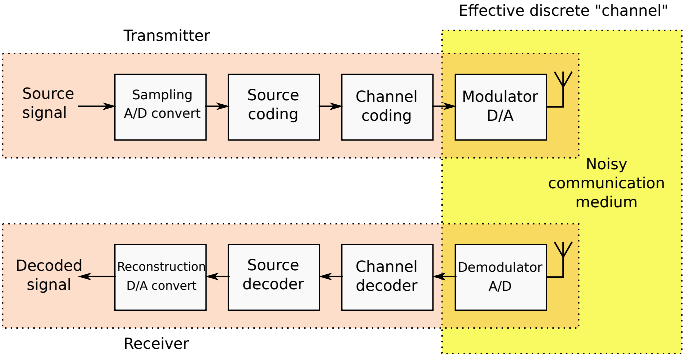

SAMSUNG
12:45
12:45 PM
Thu, March 14
London
Sunny
23°C
Google
Email
Camera
Samsung Apps
Play Store
Phone
Contacts
Messaging
Internet
Apps

Not just cellular...

- WiFi/deep space/wireless
- Wireline/optical
- Storage

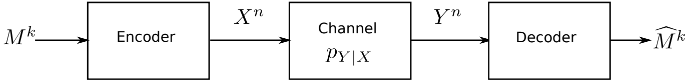
Very general!

Digital Communication system

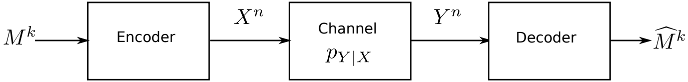


What is the maximum rate at which we can reliably communicate across a discrete memoryless channel?

Discrete memoryless channel



Discrete memoryless channel



- $M^k \sim$ iid Unif($\{0,1\}^k$)
- Memoryless channel:

$$p_{Y^n|X^n}(y^n|x^n) = \prod_{i=1}^{n} p_{Y|X}(y_i|x_i)$$

Common channels

Binary symmetric channel: BSC(p)

$\mathcal{X} = \mathcal{Y} = \{0, 1\}$, and

$$p_{Y|X}(y|x) = \begin{cases} 1 - p & \text{if } x = y \\ p & \text{if } x \neq y. \end{cases}$$

Common channels

Binary erasure channel: BEC(p)

$\mathcal{X} = \{0, 1\}$, $\mathcal{Y} = \{0, 1, e\}$

$$p_{Y|X}(y|x) = \begin{cases} p & \text{if } y = e \\ 1 - p & \text{if } x = y \\ 0 & \text{otherwise.} \end{cases}$$

Common channels

Additive white Gaussian noise (AWGN) channel

$\mathcal{X} = \mathcal{Y} = \mathbb{R}$

$$Y_i = x_i + Z_i, \qquad i = 1, 2, \ldots, n$$

where $(Z_1, \ldots, Z_n)$ are iid with $\mathcal{N}(0, \sigma^2)$ components.

Power constraint:

$$\|x^n\|^2 \stackrel{\text{def}}{=} \sum_{i=1}^{n} x_i^2 \leqslant nP$$

Common channels

Complex slow/quasi-static fading channel

$\mathcal{X} = \mathcal{Y} = \mathbb{C}$

$$Y_i = hX_i + Z_i,$$

Common channels

Fast fading channel

$$Y_i = h_i X_i + Z_i,$$

Common channels

Multiple antenna/multi-input multi-output (MIMO) channels

$\mathcal{X} = \mathbb{R}^{t_s}$, $\mathcal{Y} = \mathbb{R}^{t_r}$.

$$\underline{Y}_i = \mathsf{H}_i \underline{X}_i + \underline{Z}_i, \qquad i = 1, \ldots, n$$

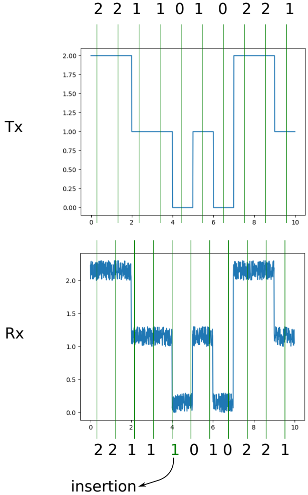
Common channels

A simple channel with memory

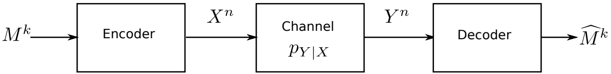
$$Y_i = a_0 X_i + a_1 X_{i-1} + \ldots + a_k X_{i-k} + Z_i$$

Common channels

Insertion/deletion channels



Channel codes



- Encoder: $f : \{0,1\}^k \to \mathcal{X}^n$
- Decoder: $g : \mathcal{Y}^n \to \{0,1\}^k$
- Rate:

$$R = \frac{k}{n}$$

- Probability of error:

$$P_e = \Pr[\widehat{M}^k \neq M^k]$$

Mutual information

$$I(X;Y) \stackrel{\text{def}}{=} \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} p_{XY}(x,y) \log_2 \frac{p_{XY}(x,y)}{p_X(x) p_Y(y)},$$

Mutual information

$$I(X;Y) \stackrel{\text{def}}{=} \sum_{x\in\mathcal{X},y\in\mathcal{Y}} p_{XY}(x,y)\log_2 \frac{p_{XY}(x,y)}{p_X(x)p_Y(y)},$$

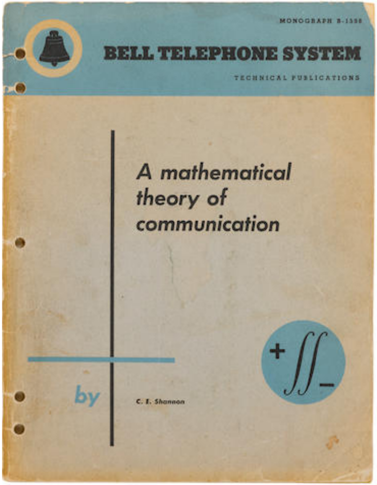
- Mutual information is symmetric
- Measures the information that X gives about Y, or Y gives about X.
- What happens if X and Y are independent?

Channel capacity

Maximum rate R for which $\lim_{n\to\infty} P_e = 0$.

Theorem (Shannon)

$$C = \max_{p_X} I(X;Y).$$

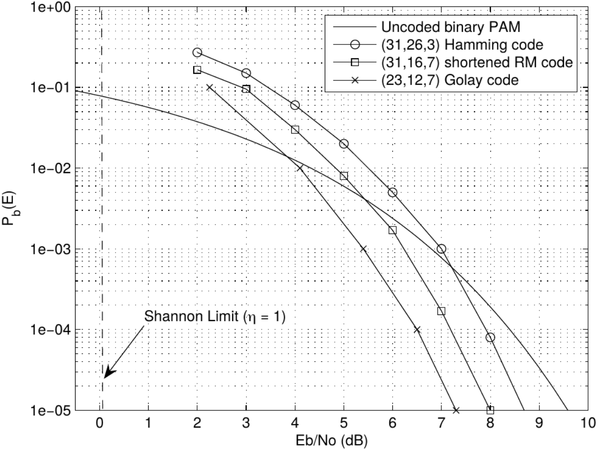

MONOGRAPH
BELL TELEPHONE SYSTEM
TECHNICAL PUBLICATIONS
A mathematical theory of communication
by
C. E. Shannon

A brief history of channel coding

The early codes

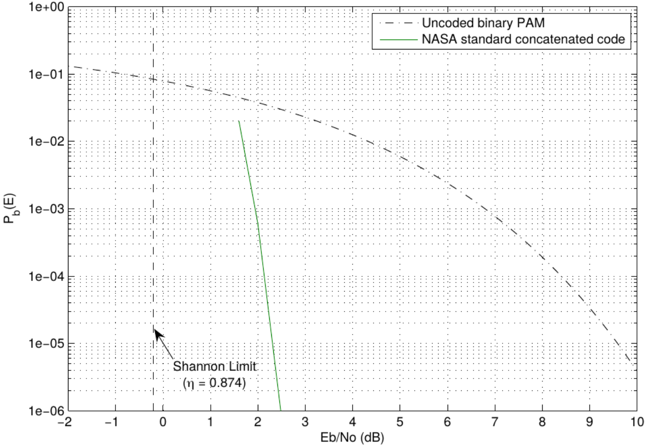
- BCH, Reed Solomon, Reed Muller codes (1960s)
- Convolutional codes (1955-1967)
- Concatenated codes (1966): deep space communication
- Trellis coded modulation (1982?): telephone lines

Performance



[0] Costello and Forney, "Channel Coding: The Road to Channel Capacity," Proceedings of the IEEE, 2007. Link: `https://arxiv.org/pdf/cs/0611112.pdf`

Performance



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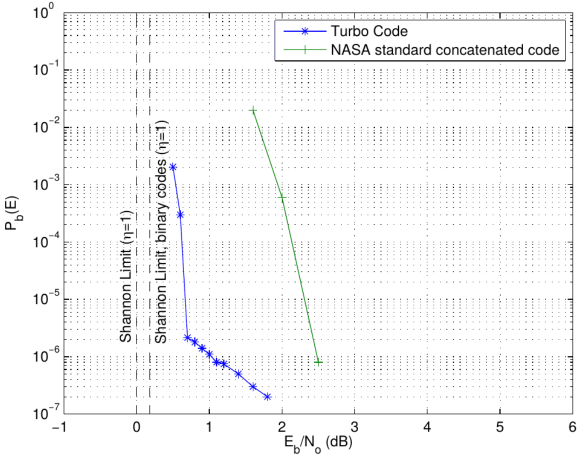
Modern codes

- Turbo codes (1993)
- LDPC codes (Gallager 1960, rediscovered 2000s)
- Polar codes (2009)

Performance



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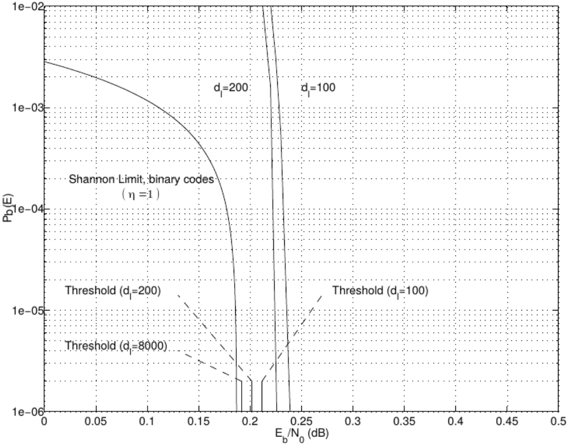
Performance



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