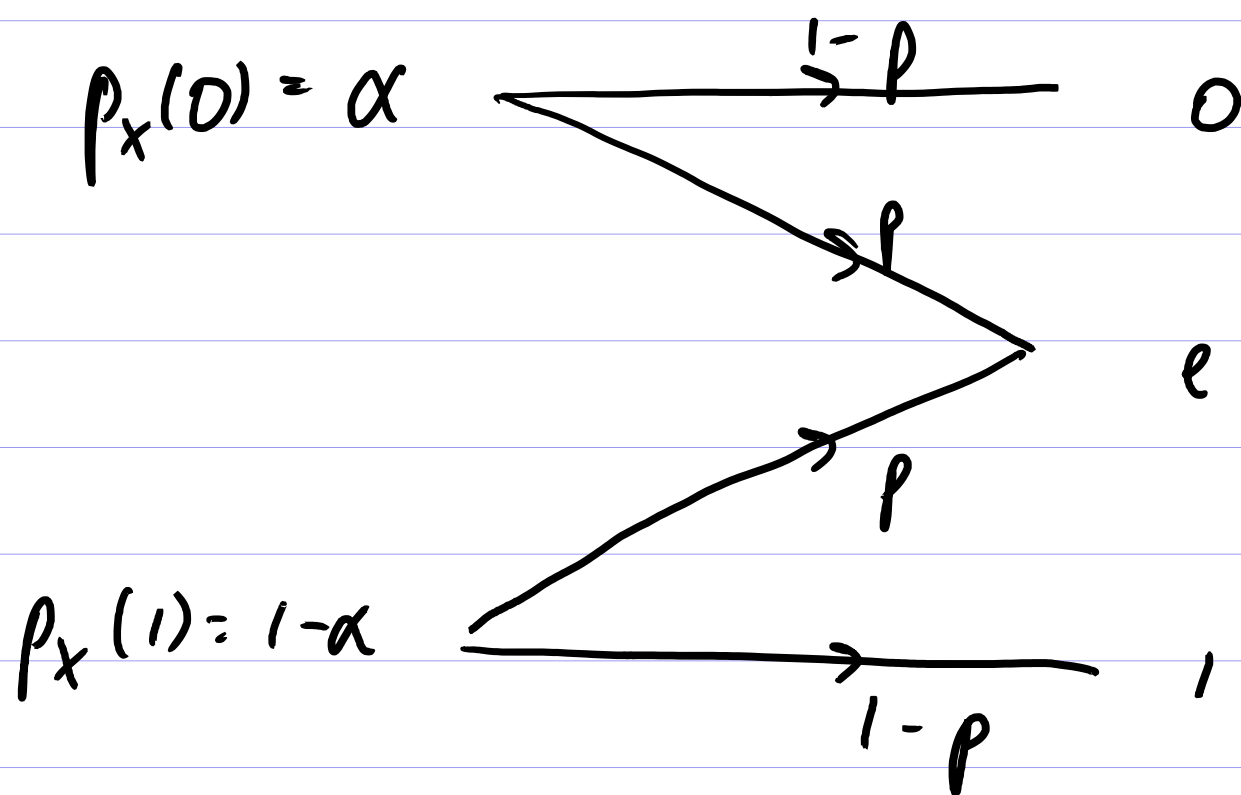


Capacity of the BEC



$$\begin{aligned}P_Y(0) &= \alpha(1-p) \\P_Y(1) &= (1-\alpha)(1-p) \\P_Y(e) &= p\end{aligned}$$

Recall that $E = \begin{cases} 1 & \text{if } Y = e \\ 0 & \text{if } Y \neq e \end{cases}$

$$P_E(1) = P_Y(e) = p.$$

$\therefore H(E) = H_2(p)$
Now compute $H(Y|E)$

$$H(Y|E) = H(Y|E=0)P_E(0) + H(Y|E=1)P_E(1)$$

$$P_{Y|E}(0|1) = 0$$

$$P_{Y|E}(1|1) = 0$$

$$P_{Y|E}(e|1) = 1$$

$$\therefore H(Y|E=1) = 0.$$

$$\begin{aligned}
 P_{Y|E}(0|0) &= \alpha \\
 P_{Y|E}(1|0) &= 1-\alpha \\
 P_{Y|E}(e|0) &= 0
 \end{aligned}$$

$$\left(\begin{aligned}
 \text{since } P_{Y|E}(0|0) &= P_{X|E}(0|0) \\
 &= \frac{P_{XE}(00)}{P_E(0)} \\
 &= \alpha \\
 \text{similar for } P_{Y|E}(1|0)
 \end{aligned} \right)$$

$$\begin{aligned}
 \therefore H(Y|E=0) &= \alpha \log_2 \frac{1}{\alpha} + (1-\alpha) \log_2 \frac{1}{1-\alpha} \\
 &= H_2(\alpha)
 \end{aligned}$$

$$H(Y|E) = 0 + (1-p) H_2(\alpha)$$

$$H(Y) = H_2(p) + (1-p) H_2(\alpha)$$

$$\begin{aligned}
 \Rightarrow \mathcal{I}(X; Y) &= H_2(p) + (1-p) H_2(\alpha) \\
 &\quad - H_2(p) \\
 &= (1-p) H_2(\alpha)
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \max_{0 \leq \alpha \leq 1} \mathcal{I}(X; Y) &= 1-p
 \end{aligned}$$