**EE2340/EE5847: Information Sciences/Information Theory** **2020**

# Homework 1: 21st Feb 2020

*Instructor: Shashank Vatedka*

**Instructions**: *You are encouraged to discuss and collaborate with your classmates. However, you* must explicitly *mention at the top of your submission who you collaborated with. Copying is NOT permitted, and solutions must be written independently and in your own words.*

*Homeworks must be submitted on Google classroom. Please scan a copy of your handwritten assignment and upload as pdf with filename <your ID>_HW<homework no>.pdf. Example: EEB19BTECH00000_HW1.pdf.*

*For programming questions, submit as separate files. Please use the naming convention <your ID>_HW<homework no>_problem<problem no>.*. Example: EEB19BTECH00000_HW1_problem1.c*

**Exercise 1.1** (Jointly typical sets)**.** Let $p_{XY}$ be any joint distribution on $\mathcal{X} \times \mathcal{Y}$. For any $\epsilon > 0$ and positive integer $n$, define the jointly typical set

$$\mathcal{T}_\epsilon^{(n)}(p_{XY}) = \left\{(x^n, y^n) \in \mathcal{X}^n \times \mathcal{Y}^n : |\mu_{x^n y^n}(a,b)/n - p_{XY}(a,b)| \leqslant \epsilon p_{XY}(a,b),\ \forall (a,b) \in \mathcal{X} \times \mathcal{Y}\right\}$$

where $\mu_{x^n y^n}(a,b)$ is the number of locations $i \in \{1, 2, \ldots, n\}$ for which $x_i = a$ and $y_i = b$.

Let $X^n, Y^n$ be jointly distributed such that $X_i, Y_i \sim p_{XY}$ for all $i$ whereas $(X_i, Y_i)$ is independent of all other $(X_j, Y_j)$ for all $j \neq i$.

1. Prove that $\lim_{n\to\infty} \Pr[(X^n, Y^n) \notin \mathcal{T}_\epsilon^{(n)}(p_{XY})] = 0$
2. Show that for all $g : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$ and all $(x^n, y^n) \in \mathcal{T}_\epsilon^{(n)}(p_{XY})$,

$$\mathbb{E}_{XY}[g(X,Y)](1-\epsilon) \leqslant \frac{1}{n}\sum_{i=1}^{n} g(x_i, y_i) \leqslant \mathbb{E}_{XY}[g(X,Y)](1+\epsilon)$$

3. Use the above to obtain upper and lower bounds on $|\mathcal{T}_\epsilon^{(n)}(p_{XY})|$.

**Exercise 1.2.** Solve exercise 3.13 in the second edition of the book by Cover and Thomas.

**Exercise 1.3** (Shannon code)**.** The Shannon code is a variable length code which assigns codeword length of $\lceil \log_2 \frac{1}{p(a)} \rceil$ to a symbol with probability $p(a)$. The codewords are generated using the complete binary tree, as you saw in the proof of Kraft's inequality. Attached is a file `randomsequence.txt` with characters randomly drawn from the set $\{a, b, c, d, e\}$. You must write a program to compress and decompress this file.

1. Write a program to compute the empirical type of the file, i.e., the fraction of occurrence of various symbols in the file. Also compute the entropy.
2. Using the above type, design the Shannon code for this file.
3. Use the designed code to compress the attached file.
4. Decode the compressed file, and verify that you get back the original file
5. Find the length of the compressed sequence, and compare this with the entropy.