Data-flow Analysis

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Data-flow analysis

- These are techniques that derive information about the flow of data along program execution paths
- An *execution path* (or *path*) from point p_1 to point p_n is a sequence of points $p_1, p_2, ..., p_n$ such that for each $i = 1, 2, ..., n - 1$, either
  1. p_i is the point immediately preceding a statement and p_{i+1} is the point immediately following that same statement, or
  2. p_i is the end of some block and p_{i+1} is the beginning of a successor block
- In general, there is an infinite number of paths through a program and there is no bound on the length of a path
- Program analyses summarize all possible program states that can occur at a point in the program with a finite set of facts
- No analysis is necessarily a perfect representation of the state

Uses of Data-flow Analysis

- Program debugging
  - Which are the definitions (of variables) that *may* reach a program point? These are the *reaching definitions*
- Program optimizations
  - Constant folding
  - Copy propagation
  - Common sub-expression elimination etc.

Data-Flow Analysis Schema

- A *data-flow value* for a program point represents an abstraction of the set of all possible program states that can be observed for that point
- The set of all possible data-flow values is the *domain* for the application under consideration
  - Example: for the *reaching definitions* problem, the domain of data-flow values is the set of all subsets of of definitions in the program
  - A particular data-flow value is a set of definitions
- $IN[s]$ and $OUT[s]$: data-flow values *before* and *after* each statement s
- The *data-flow problem* is to find a solution to a set of constraints on $IN[s]$ and $OUT[s]$, for all statements s

Data-Flow Analysis Schema (2)

- Two kinds of constraints
  - Those based on the semantics of statements (*transfer functions*)
  - Those based on flow of control
- A DFA schema consists of
  - A control-flow graph
  - A direction of data-flow (forward or backward)
  - A set of data-flow values
  - A confluence operator (normally set union or intersection)
  - Transfer functions for each block
- We always compute *safe* estimates of data-flow values
- A decision or estimate is *safe* or *conservative*, if it never leads to a change in what the program computes (after the change)
- These safe values may be either subsets or supersets of actual values, based on the application

The Reaching Definitions Problem

- We *kill* a definition of a variable *a*, if between two points along the path, there is an assignment to *a*
- A definition *d* reaches a point *p*, if there is a path from the point immediately following *d* to *p*, such that *d* is not *killed* along that path
- Unambiguous and ambiguous definitions of a variable

    a := b+c

  (unambiguous definition of 'a')

    ...

    *p := d

  (ambiguous definition of 'a', if 'p' may point to variables other than 'a' as well; hence does not kill the above definition of 'a')

    ...

    a := k-m

  (unambiguous definition of 'a'; kills the above definition of 'a')

The Reaching Definitions Problem(2)

- We compute supersets of definitions as *safe* values
- It is safe to assume that a definition reaches a point, even if it does not.
- In the following example, we assume that both `a=2` and `a=4` reach the point after the complete if-then-else statement, even though the statement `a=4` is not reached by control flow

```
if (a==b) a=2; else if (a==b) a=4;
```

The Reaching Definitions Problem (3)

- The data-flow equations (constraints)

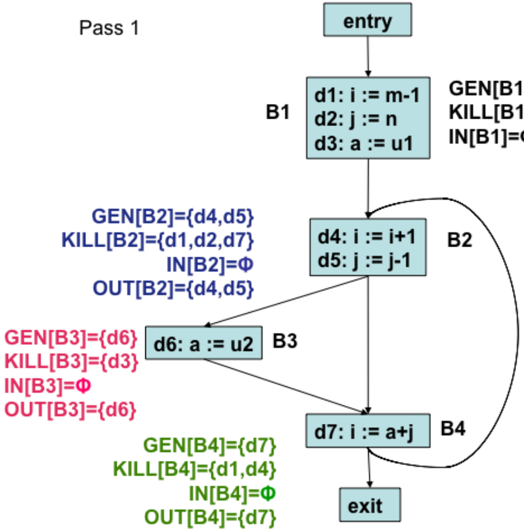
$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

$$IN[B] = \phi, \text{ for all } B \text{ (initialization only)}$$

- If some definitions reach B_1 (entry), then $IN[B_1]$ is initialized to that set
- Forward flow DFA problem (since $OUT[B]$ is expressed in terms of $IN[B]$), confluence operator is $\cup$
- $GEN[B]$ = set of all definitions inside B that are "visible" immediately after the block - *downwards exposed* definitions
- $KILL[B]$ = union of the definitions in all the basic blocks of the flow graph, that are killed by individual statements in B

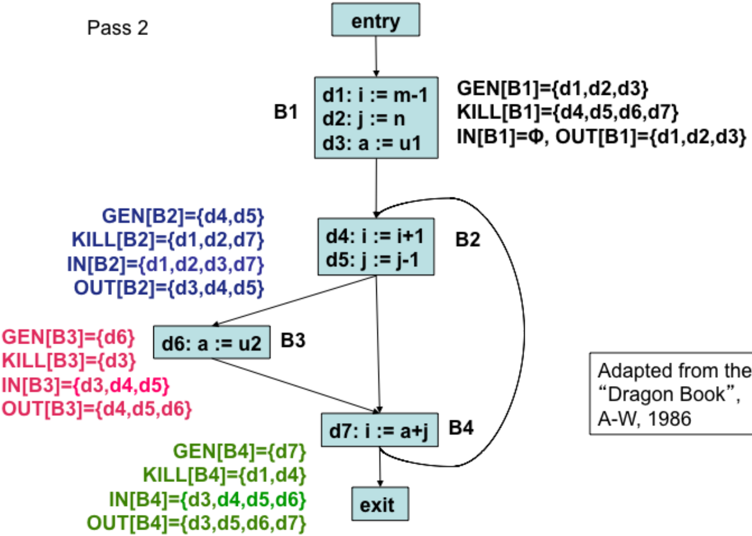
Reaching Definitions Analysis: An Example - Pass 1



Adapted from the "Dragon Book", A-W, 1986

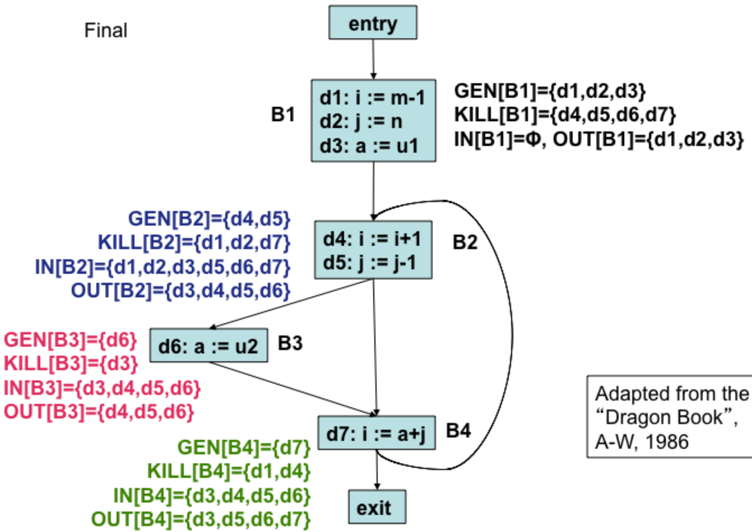
Reaching Definitions Analysis: An Example - Pass 2

Pass 2



Adapted from the “Dragon Book”, A-W, 1986

Reaching Definitions Analysis: An Example - Final



Adapted from the "Dragon Book", A-W, 1986

An Iterative Algorithm for Computing Reaching Definitions

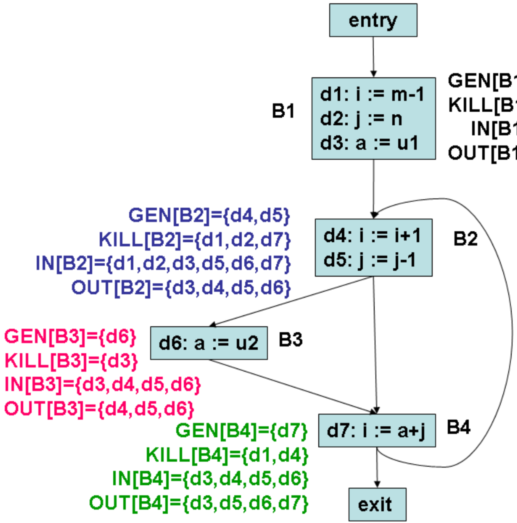
for each block B do { $IN[B] = \phi$; $OUT[B] = GEN[B]$; }
$change = true$;
while $change$ do { $change = false$;
  for each block B do {

$$\begin{aligned} IN[B] &= \bigcup_{P \ a \ predecessor \ of \ B} OUT[P]; \\ oldout &= OUT[B]; \\ OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]); \end{aligned}$$

    if ($OUT[B] \neq oldout$) $change = true$;
  }
}

- GEN, $KILL$, IN, and OUT are all represented as bit vectors with one bit for each definition in the flow graph

Reaching Definitions: Bit Vector Representation



Final dataflow value sets shown in bit vector format

	d1	d2	d3	d4	d5	d6	d7
GEN[B1]=	1	1	1	0	0	0	0
KILL[B1]=	0	0	0	1	1	1	1
IN[B1]=	0	0	0	0	0	0	0
OUT[B1]=	1	1	1	0	0	0	0

Adapted from the "Dragon Book", A-W, 1986

Use-Definition Chains (u-d chains)

- Reaching definitions may be stored as u-d chains for convenience
- A u-d chain is a list of a use of a variable and all the definitions that reach that use
- u-d chains may be constructed once reaching definitions are computed
- **case 1**: If use $u1$ of a variable b in block B is preceded by no unambiguous definition of b, then attach all definitions of b in $IN[B]$ to the u-d chain of that use $u1$ of b
- **case 2**: If any unambiguous definition of b preceeds a use of b, then *only that definition* is on the u-d chain of that use of b
- **case 3**: If any ambiguous definitions of b precede a use of b, then each such definition for which no unambiguous definition of b lies between it and the use of b, are on the u-d chain for this use of b

Use-Definition Chain Construction

IN[B]

B

no unambiguous def. of ‘b’

:= b (use u1)

attach def of ‘b’ in IN[B] to u-d chain of use u1

B

b:= (def d1)

no other unambiguous def. of ‘b’ here

:= b (use u1)

attach def d1 alone to use u1

B

b := (def d1)

...

*p := (ambiguous definition of ‘b’,d2)

...

no other unambiguous def. of ‘b’ here

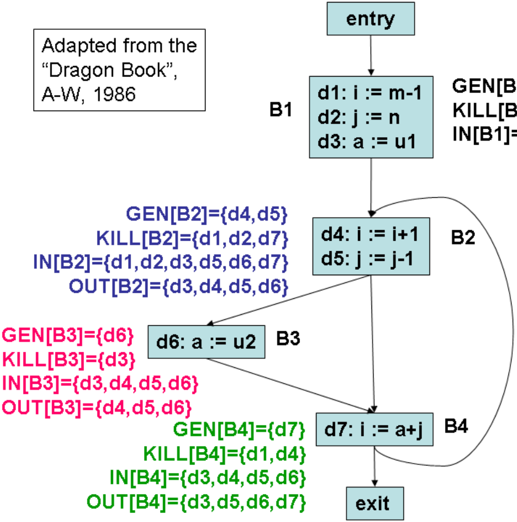
:= b (use u1)

attach both d1 and d2 to use u1

Three cases while constructing u-d chains from the reaching definitions

Use-Definition Chain Example

Adapted from the "Dragon Book", A-W, 1986



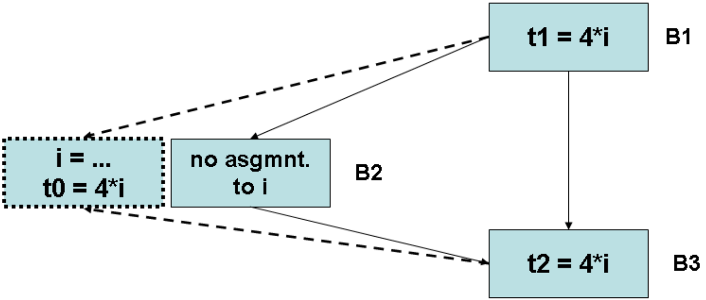
use	u-d chain
(i,d4)	(d1,d7)
(j,d5)	(d2,d5)
(a,d7)	(d3,d6)
(j,d7)	(d5)

Available Expression Computation

- Sets of expressions constitute the domain of data-flow values
- Forward flow problem
- Confluence operator is $\cap$
- An expression $x + y$ is *available* at a point p, if every path (not necessarily cycle-free) from the initial node to p evaluates $x + y$, and after the last such evaluation, prior to reaching p, there are no subsequent assignments to x or y
- A block *kills* $x + y$, if it assigns (or may assign) to x or y and does not subsequently recompute $x + y$.
- A block *generates* $x + y$, if it definitely evaluates $x + y$, and does not subsequently redefine x or y

Available Expression Computation(2)

- Useful for global common sub-expression elimination
- $4 * i$ is a CSE in $B3$, if it is available at the entry point of $B3$ *i.e.,* if i is not assigned a new value in $B2$ or $4 * i$ is recomputed after i is assigned a new value in $B2$ (as shown in the dotted box)



Available Expression Computation (3)

- The data-flow equations

$$
\begin{aligned}
IN[B] &= \bigcap_{P \text{ is a predecessor of } B} OUT[P], \ B \text{ not initial} \\
OUT[B] &= e\_gen[B] \bigcup (IN[B] - e\_kill[B]) \\
IN[B1] &= \phi \\
IN[B] &= U, \ \text{for all } B \neq B1 \ (\text{initialization only})
\end{aligned}
$$

- $B1$ is the intial or entry block and is special because nothing is available when the program begins execution
- $IN[B1]$ is always ϕ
- U is the universal set of all expressions
- Initializing $IN[B]$ to ϕ for all $B \neq B1$, is restrictive

Computing e_gen and e_kill

- For statements of the form $x = a$, step 1 below does not apply
- The set of all expressions appearing as the RHS of assignments in the flow graph is assumed to be available and is represented using a hash table and a bit vector

e_gen[q] = A

q ▪
x = y + z
p ▪

Computing e_gen[p]

1. A = A **U** {y+z}
2. A = A – {all expressions involving x}
3. e_gen[p] = A

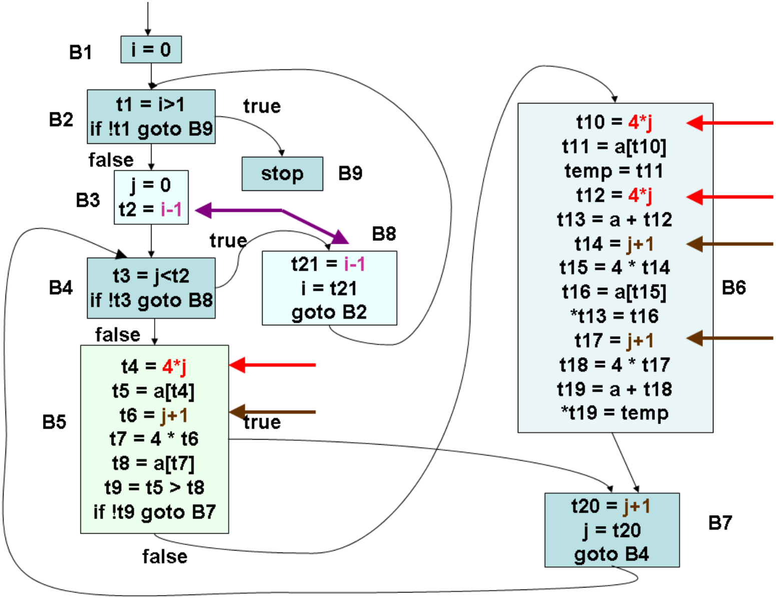
e_kill[q] = A

q ▪
x = y + z
p ▪

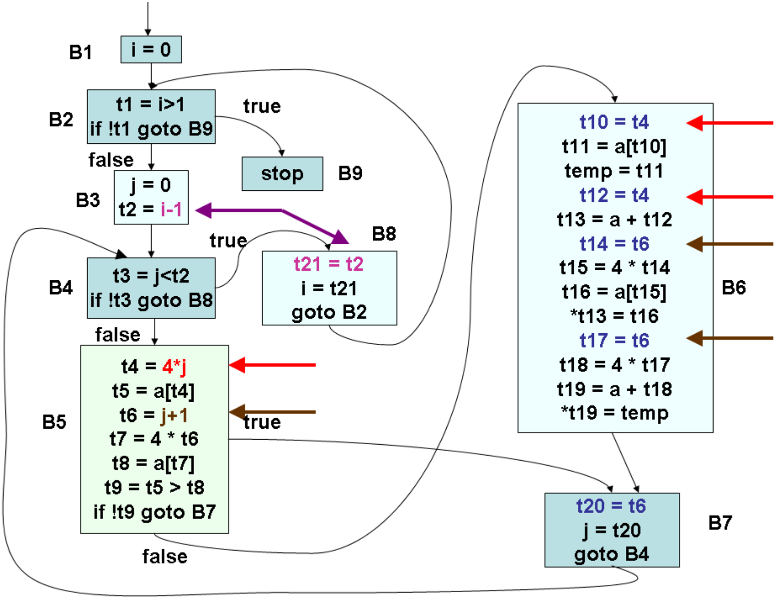
Computing e_kill[p]

1. A = A – {y+z}
2. A = A **U** {all expressions involving x}
3. e_kill[p] = A

Available Expression Computation - An Example



Available Expression Computation - An Example (2)



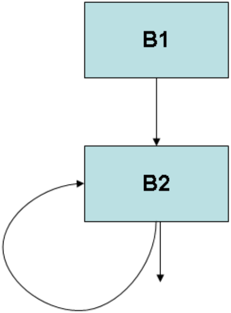
An Iterative Algorithm for Computing Available Expressions

for each block $B \neq B1$ do $\{OUT[B] = U - e\_kill[B];\}$
/* You could also do $IN[B] = U$;*/
/* In such a case, you must also interchange the order of */
/* $IN[B]$ and $OUT[B]$ equations below */
$change = true$;
while $change$ do { $change = false$;
  for each block $B \neq B1$ do {

$$IN[B] = \bigcap_{P\ a\ predecessor\ of\ B} OUT[P];$$
$$oldout = OUT[B];$$
$$OUT[B] = e\_gen[B] \bigcup (IN[B] - e\_kill[B]);$$

    if $(OUT[B] \neq oldout)$ $change = true$;
  }
}

Initializing IN[B] to ϕ for all B can be restrictive



Let e_gen[B2] be G and e_kill[B2] be K

IN[B2] = OUT[B1] ∩ OUT[B2]

OUT[B2] = G U (IN[B2] – K)

IN^0[B2]=Φ, OUT^1[B2]=G

IN^1[B2]=OUT[B1] ∩ G

OUT^2[B2]=G U ((OUT[B1] ∩ G) – K)

= G U G = G

Note that (OUT[B1] ∩ G) is always smaller than G

IN^0[B2]= ***U***, OUT^1[B2]= ***U*** - K

IN^1[B2]=OUT[B1] ∩ (***U*** – K)

= OUT[B1] - K

OUT^2[B2]=G U ((OUT[B1] - K) – K)

= G U (OUT[B1] - K)

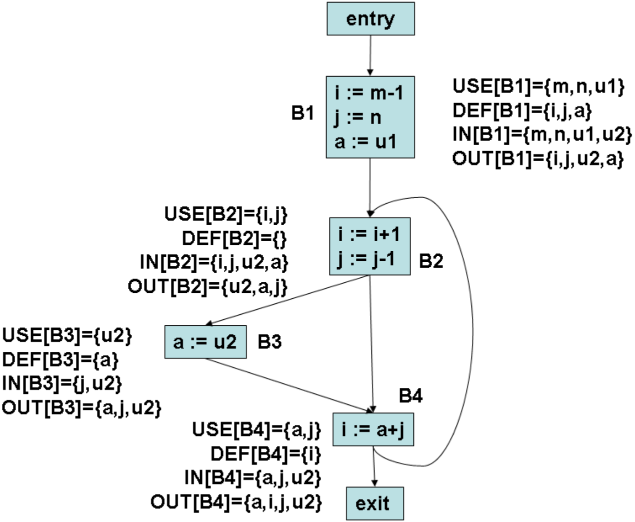
This set OUT[B2] is larger and more intuitive, but still correct

Live Variable Analysis

- The variable x is *live* at the point p, if the value of x at p could be used along some path in the flow graph, starting at p; otherwise, x is *dead* at p
- Sets of variables constitute the domain of data-flow values
- Backward flow problem, with confluence operator $\bigcup$
- $IN[B]$ is the set of variables live at the beginning of B
- $OUT[B]$ is the set of variables live just after B
- $DEF[B]$ is the set of variables definitely assigned values in B, prior to any use of that variable in B
- $USE[B]$ is the set of variables whose values may be used in B prior to any definition of the variable

$$
\begin{aligned}
OUT[B] &= \bigcup_{S\ is\ a\ successor\ of\ B} IN[S] \\
IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B]) \\
IN[B] &= \phi, \textit{ for all } B \textit{ (initialization only)}
\end{aligned}
$$

Live Variable Analysis: An Example



Definition-Use Chains (d-u chains)

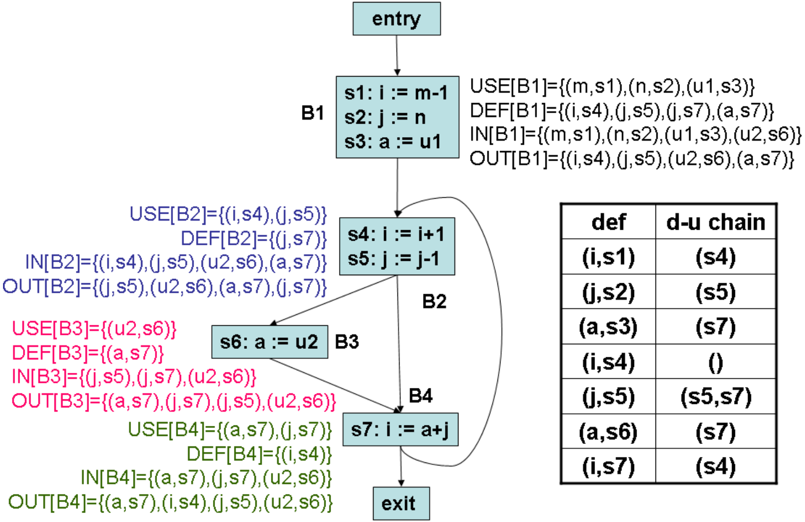
- For each definition, we wish to attach the statement numbers of the uses of that definition
- Such information is very useful in implementing register allocation, loop invariant code motion, etc.
- This problem can be transformed to the data-flow analysis problem of computing for a point p, the set of uses of a variable (say x), such that there is a path from p to the use of x, that does not redefine x.
- This information is represented as sets of (x, s) pairs, where x is the variable used in statement s
- In live variable analysis, we need information on whether a variable is used later, but in (x, s) computation, we also need the statment numbers of the uses
- The data-flow equations are similar to that of LV analysis
- Once $IN[B]$ and $OUT[B]$ are computed, d-u chains can be computed using a method similar to that of u-d chains

Data-flow Analysis for (x,s) pairs

- Sets of pairs (x,s) constitute the domain of data-flow values
- Backward flow problem, with confluence operator $\bigcup$
- $USE[B]$ is the set of pairs (x, s), such that s is a statement in B which uses variable x and such that no prior definition of x occurs in B
- $DEF[B]$ is the set of pairs (x, s), such that s is a statement which uses x, s is *not in B*, and B contains a definition of x
- $IN[B]$ ($OUT[B]$, resp.) is the set of pairs (x, s), such that statement s uses variable x and the value of x at $IN[B]$ ($OUT[B]$, resp.) has not been modified along the path from $IN[B]$ ($OUT[B]$, resp.) to s

$$\begin{aligned} OUT[B] &= \bigcup_{S \text{ is a successor of } B} IN[S] \\ IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B]) \\ IN[B] &= \phi, \text{ for all } B \text{ (initialization only)} \end{aligned}$$

Definition-Use Chain Example



def	d-u chain
(i,s1)	(s4)
(j,s2)	(s5)
(a,s3)	(s7)
(i,s4)	()
(j,s5)	(s5,s7)
(a,s6)	(s7)
(i,s7)	(s4)

Definition-Use Chain Construction

B

b := (def d1)
no
unambiguous
def. of ‘b’

↓

OUT[B]

attach to du-chain of d1, stmts s_i of all use pairs (b,s_i) in OUT[B]

B

b:= (def d1)

no other unambiguous def. of ‘b’ here

:= b (use u1)

attach use u1 to du-chain of def d1

B

b := (def d1)
...
***p := (ambiguous definition of ‘b’,d2)**
...
no other unambiguous def. of ‘b’ here

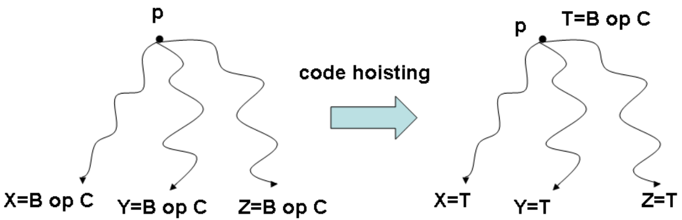
:= b (use u1)

attach use u1 to du-chains of both def d1 and def d2

Three cases while constructing d-u chains from the (x,s) pairs

Very Busy Expressions *or* Anticipated Expressions

- An expression *B op C* is very busy or anticipated at a point *p*, if along every path from *p*, we come to a computation of *B op C* before any computation of *B* or *C*
- Useful in code hoisting and partial redundancy elimination
- Code hoisting does not reduce time, but reduces space
- We must make sure that no use of *B op C* (from X,Y, or Z below) has any definition of *B* or *C* reaching it without passing through *p*

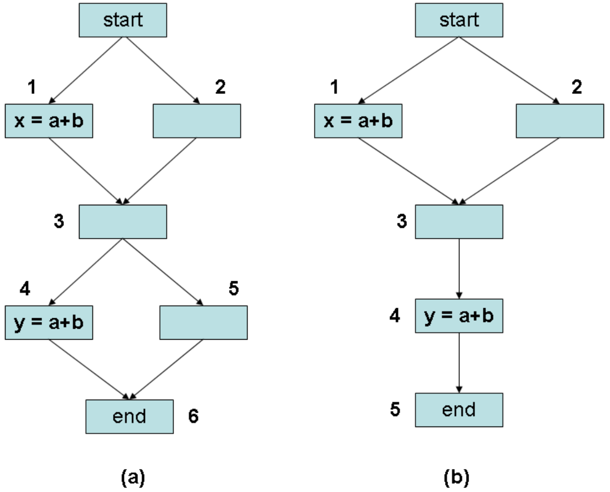


Very Busy Expressions *or* Anticipated Expressions (2)

- Sets of expressions constitute the domain of data-flow values
- Backward flow analysis with $\bigcap$ as confluence operator
- $V\_USE[n]$ is the set of expressions $B\ op\ C$ computed in n with no prior definition of B or C in n
- $V\_DEF[n]$ is the set of expressions $B\ op\ C$ in U (the universal set of expressions) for which either B or C is defined in n, prior to any computation of $B\ op\ C$

$$
\begin{aligned}
OUT[n] &= \bigcap_{S\ is\ a\ successor\ of\ n} IN[S] \\
IN[n] &= V\_USE[n] \bigcup (OUT[n] - V\_DEF[n]) \\
IN[n] &= U, \mathit{for\ all\ n\ (initialization\ only)}
\end{aligned}
$$

Anticipated Expressions - An Example



a+b is anticipated at: entry to 1 and 4
a+b is not anticipated at: all other points

a+b is anticipated at all points, except at exit of 4 and entry of 5

The Reaching Definitions Problem

- Domain of data-flow values: sets of definitions
- Direction: Forwards
- Confluence operator: $\cup$
- Initialization: $IN[B] = \phi$
- Equations:

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Data-Flow Problems: A Summary - 2

The Available Expressions Problem

- Domain of data-flow values: sets of expressions
- Direction: Forwards
- Confluence operator: $\cap$
- Initialization: $IN[B] = U$
- Equations:

$$
\begin{aligned}
IN[B] &= \bigcap_{P \text{ is a predecessor of } B} OUT[P] \\
OUT[B] &= e\_gen[B] \bigcup (IN[B] - e\_kill[B]) \\
IN[B1] &= \phi
\end{aligned}
$$

The Live Variable Analysis Problem

- Domain of data-flow values: sets of variables
- Direction: backwards
- Confluence operator: $\cup$
- Initialization: $IN[B] = \phi$
- Equations:

$$
\begin{aligned}
OUT[B] &= \bigcup_{S \text{ is a successor of } B} IN[S] \\
IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B])
\end{aligned}
$$

The Anticipated Expressions (Very Busy Expressions) Problem

- Domain of data-flow values: sets of expressions
- Direction: backwards
- Confluence operator: $\cap$
- Initialization: $IN[B] = U$
- Equations:

$$\begin{aligned} OUT[B] &= \bigcap_{S\ is\ a\ successor\ of\ B} IN[S] \\ IN[B] &= V\_USE[B] \bigcup (OUT[B] - V\_DEF[B]) \end{aligned}$$