Introduction to Machine-Independent Optimizations - 2

Data-Flow Analysis

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NPTEL Course on Principles of Compiler Design

Outline of the Lecture

- What is code optimization? (in part 1)
- Illustrations of code optimizations (in part 1)
- Examples of data-flow analysis
- Fundamentals of control-flow analysis
- Algorithms for two machine-independent optimizations
- SSA form and optimizations

Data-Flow Analysis Schema

- A *data-flow value* for a program point represents an abstraction of the set of all possible program states that can be observed for that point
- The set of all possible data-flow values is the *domain* for the application under consideration
  - Example: for the *reaching definitions* problem, the domain of data-flow values is the set of all subsets of of definitions in the program
  - A particular data-flow value is a set of definitions
- $IN[s]$ and $OUT[s]$: data-flow values *before* and *after* each statement s
- The *data-flow problem* is to find a solution to a set of constraints on $IN[s]$ and $OUT[s]$, for all statements s

Data-Flow Analysis Schema (2)

- Two kinds of constraints
  - Those based on the semantics of statements (*transfer functions*)
  - Those based on flow of control
- A DFA schema consists of
  - A control-flow graph
  - A direction of data-flow (forward or backward)
  - A set of data-flow values
  - A confluence operator (usually set union or intersection)
  - Transfer functions for each block
- We always compute *safe* estimates of data-flow values
- A decision or estimate is *safe* or *conservative*, if it never leads to a change in what the program computes (after the change)
- These safe values may be either subsets or supersets of actual values, based on the application

The Reaching Definitions Problem

- We *kill* a definition of a variable *a*, if between two points along the path, there is an assignment to *a*
- A definition *d* reaches a point *p*, if there is a path from the point immediately following *d* to *p*, such that *d* is not *killed* along that path
- Unambiguous and ambiguous definitions of a variable

  a := b+c

  (unambiguous definition of 'a')

  ...

  *p := d

  (ambiguous definition of 'a', if 'p' may point to variables other than 'a' as well; hence does not kill the above definition of 'a')

  ...

  a := k-m

  (unambiguous definition of 'a'; kills the above definition of 'a')

The Reaching Definitions Problem(2)

- We compute supersets of definitions as *safe* values
- It is safe to assume that a definition reaches a point, even if it does not.
- In the following example, we assume that both `a=2` and `a=4` reach the point after the complete if-then-else statement, even though the statement `a=4` is not reached by control flow

```
if (a==b) a=2; else if (a==b) a=4;
```

The Reaching Definitions Problem (3)

- The data-flow equations (constraints)

$$
\begin{aligned}
IN[B] &= \bigcup_{P \text{ is a predecessor of } B} OUT[P] \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]) \\
IN[B] &= \phi, \textit{for all } B \ (\textit{initialization only})
\end{aligned}
$$

- If some definitions reach B_1 (entry), then $IN[B_1]$ is initialized to that set
- Forward flow DFA problem (since $OUT[B]$ is expressed in terms of $IN[B]$), confluence operator is $\cup$
  - Direction of flow does not imply traversing the basic blocks in a particular order
  - The final result does not depend on the order of traversal of the basic blocks

The Reaching Definitions Problem (4)

- $GEN[B]$ = set of all definitions inside B that are “visible” immediately after the block - *downwards exposed* definitions
  - If a variable x has two or more defintions in a basic block, then only the last definition of x is downwards exposed; all others are not visible outside the block
- $KILL[B]$ = union of the definitions in all the basic blocks of the flow graph, that are killed by individual statements in B
  - If a variable x has a definition d_i in a basic block, then d_i kills all the definitions of the variable x in the program, except d_i

Reaching Definitions Analysis: GEN and KILL

In other blocks:
d5: b = a+4
d6: f = e+c
d7: e = b+d
d8: d = a+b
d9: a = c+f
d10: c = e+a

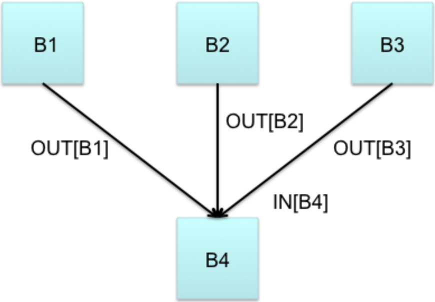
B:

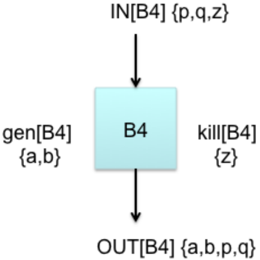
d1: a = f + 1
d2: b = a + 7
d3: c = b + d
d4: a = d + c

Set of all definitions = {d1,d2,d3,d4,d5,d6,d7,d8,d9,10}

GEN[B] = {d2,d3,d4}
KILL[B] = {d4,d9,d5,d10,d1}

Reaching Definitions Analysis: DF Equations





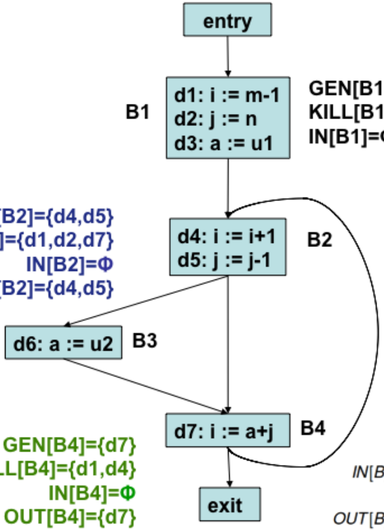
IN[B4] = OUT[B1] U OUT[B2] U OUT[B3]

$$
\begin{aligned}
IN[B] &= \bigcup_{P \text{ is a predecessor of } B} OUT[P] \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B])
\end{aligned}
$$

OUT[B4] = gen[B4] U (IN[B4] – kill[B4])

Reaching Definitions Analysis: An Example - Pass 1

Pass 1



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]=Φ
OUT[B2]={d4,d5}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]=Φ
OUT[B3]={d6}

GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]=Φ
OUT[B4]={d7}

Adapted from the "Dragon Book", A-W, 1986

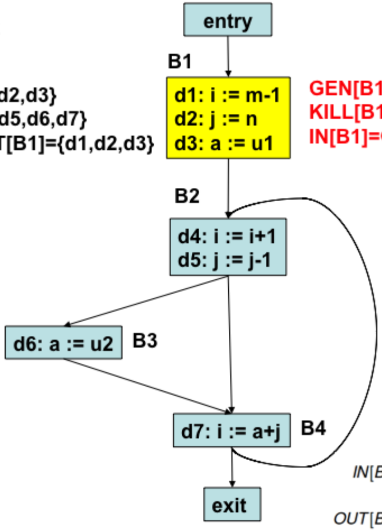
$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Pass 2.1

Pass 2

GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

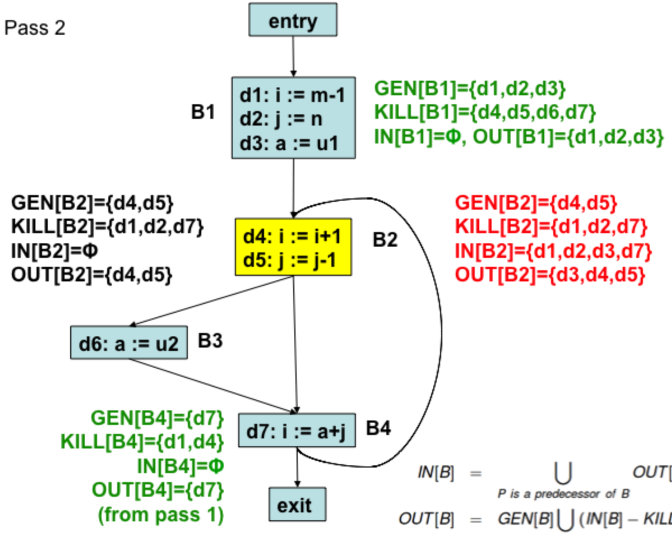


GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

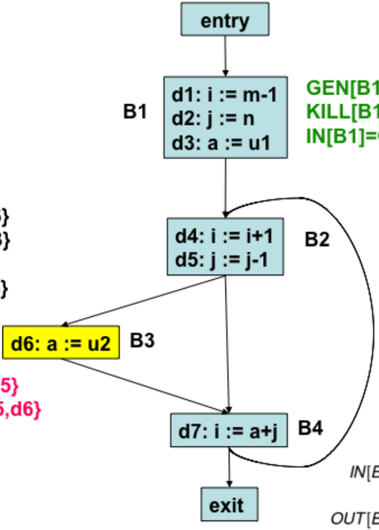
Reaching Definitions Analysis: An Example - Pass 2.2



$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Pass 2



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

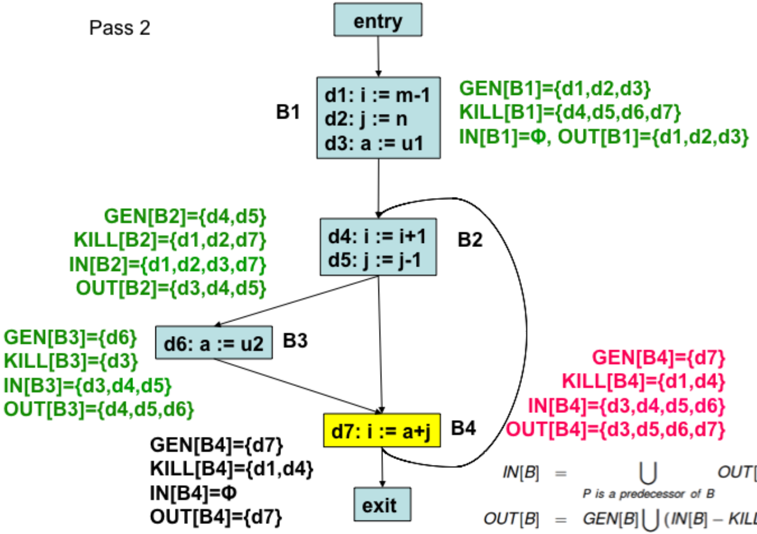
GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]=Φ
OUT[B3]={d6}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d7}
OUT[B2]={d3,d4,d5}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5}
OUT[B3]={d4,d5,d6}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

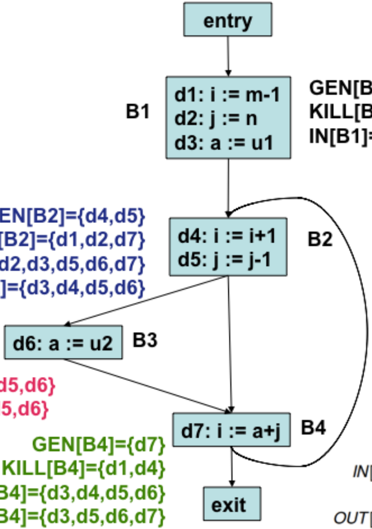


$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Final

Final



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d5,d6,d7}
OUT[B2]={d3,d4,d5,d6}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5,d6}
OUT[B3]={d4,d5,d6}

GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]={d3,d4,d5,d6}
OUT[B4]={d3,d5,d6,d7}

Adapted from the "Dragon Book", A-W, 1986

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

An Iterative Algorithm for Computing Reaching Def.

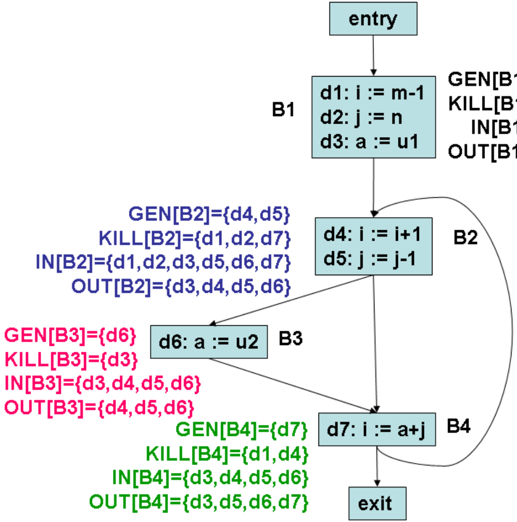
for each block B do { $IN[B] = \phi$; $OUT[B] = GEN[B]$; }
$change = true$;
while $change$ do { $change = false$;
for each block B do {

$$\begin{aligned} IN[B] &= \bigcup_{P \text{ a predecessor of } B} OUT[P]; \\ oldout &= OUT[B]; \\ OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]); \end{aligned}$$

if ($OUT[B] \neq oldout$) $change = true$;
}
}

- GEN, $KILL$, IN, and OUT are all represented as bit vectors with one bit for each definition in the flow graph

Reaching Definitions: Bit Vector Representation



Final dataflow value sets shown in bit vector format

	d1	d2	d3	d4	d5	d6	d7
GEN[B1]=	1	1	1	0	0	0	0
KILL[B1]=	0	0	0	1	1	1	1
IN[B1]=	0	0	0	0	0	0	0
OUT[B1]=	1	1	1	0	0	0	0

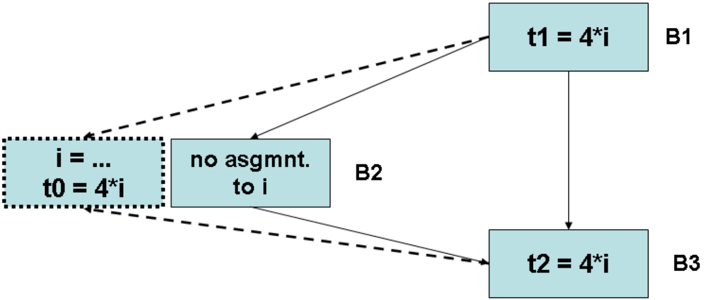
Adapted from the "Dragon Book", A-W, 1986

Available Expression Computation

- Sets of expressions constitute the domain of data-flow values
- Forward flow problem
- Confluence operator is $\cap$
- An expression $x + y$ is *available* at a point p, if every path (not necessarily cycle-free) from the initial node to p evaluates $x + y$, and after the last such evaluation, prior to reaching p, there are no subsequent assignments to x or y
- A block *kills* $x + y$, if it assigns (or may assign) to x or y and does not subsequently recompute $x + y$.
- A block *generates* $x + y$, if it definitely evaluates $x + y$, and does not subsequently redefine x or y

Available Expression Computation(2)

- Useful for global common sub-expression elimination
- $4 * i$ is a CSE in $B3$, if it is available at the entry point of $B3$ *i.e.,* if i is not assigned a new value in $B2$ or $4 * i$ is recomputed after i is assigned a new value in $B2$ (as shown in the dotted box)



Computing e_gen and e_kill

- For statements of the form $x = a$, step 1 below does not apply
- The set of all expressions appearing as the RHS of assignments in the flow graph is assumed to be available and is represented using a hash table and a bit vector

e_gen[q] = A **q** ▪
x = y + z
p ▪

Computing e_gen[p]

1. A = A **U** {y+z}
2. A = A – {all expressions involving x}
3. e_gen[p] = A

e_kill[q] = A **q** ▪
x = y + z
p ▪

Computing e_kill[p]

1. A = A – {y+z}
2. A = A **U** {all expressions involving x}
3. e_kill[p] = A