Syntax Analysis:

Context-free Grammars, Pushdown Automata and Parsing
Part - 5

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NPTEL Course on Principles of Compiler Design

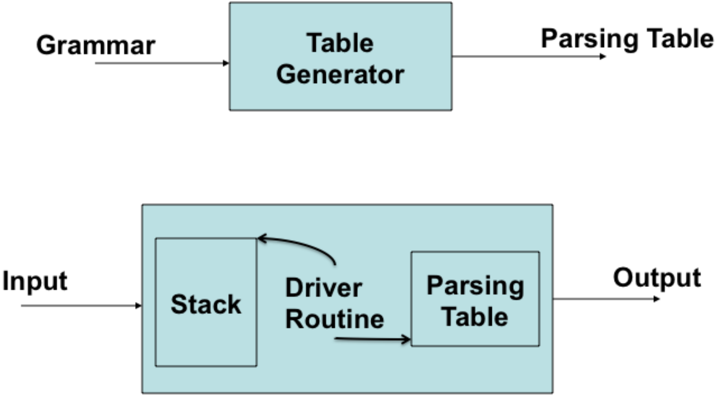
Outline of the Lecture

- What is syntax analysis? (covered in lecture 1)
- Specification of programming languages: context-free grammars (covered in lecture 1)
- Parsing context-free languages: push-down automata (covered in lectures 1 and 2)
- Top-down parsing: LL(1) parsing (covered in lectures 2 and 3)
- Recursive-descent parsing (covered in lecture 4)
- Bottom-up parsing: LR-parsing

LR Parsing

- LR(k) - *L*eft to right scanning with *R*ightmost derivation in reverse, *k* being the number of lookahead tokens
    - $k = 0, 1$ are of practical interest
- LR parsers are also automatically generated using parser generators
- LR grammars are a subset of CFGs for which LR parsers can be constructed
- LR(1) grammars can be written quite easily for practically all programming language constructs for which CFGs can be written
- LR parsing is the most general non-backtracking shift-reduce parsing method (known today)
- LL grammars are a strict subset of LR grammars - an LL(k) grammar is also LR(k), but not vice-versa

LR Parser Generation

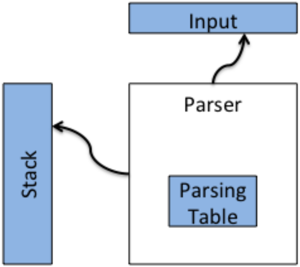


LR Parser Generator

LR Parser Configuration

- A configuration of an LR parser is:
  $(s_0 X_1 s_2 X_2 ... X_m s_m, \quad a_i a_{i+1} ... a_n \$)$, where,
  stack **unexpended input**
  $s_0, s_1, ..., s_m$, are the states of the parser, and $X_1, X_2, ..., X_m$, are grammar symbols (terminals or nonterminals)
- Starting configuration of the parser: $(s_0, a_1 a_2 ... a_n \$)$, where, s_0 is the initial state of the parser, and $a_1 a_2 ... a_n$ is the string to be parsed
- Two parts in the parsing table: *ACTION* and *GOTO*
  - The *ACTION* table can have four types of entries: **shift, reduce, accept**, or **error**
  - The *GOTO* table provides the next state information to be used after a *reduce* move

LR Parsing Algorithm



Initial configuration: Stack = *state 0*, Input = *w$*,
a = first input symbol;
repeat {
  let *s* be the top stack state;
  let *a* be the next input symbol;
  if (*ACTION[s, a] == shift p*) {
    push *a* and *p* onto the stack (in that order);
    advance input pointer;
  } else if (*ACTION[s,a] == reduce A* → α) then {
      pop *2*|α|* symbols off the stack;
      let *s'* be the top of stack state now;
      push *A* and *GOTO[s', A]* onto the stack
      (in that order);
    } else if (*ACTION[s, a] == accept*) break;
      /* parsng is over */
      else *error()*;
} until true; /* for ever */

LR Parsing Example 1 - Parsing Table

STATE	ACTION				GOTO		
	a	b	c	$	S	A	B
0	S2		S3		1		
1				R1 acc			
2	S2	S6	S3		8	4	
3	R3	R3	R3	R3			
4	S2		S3		5		
5	R2	R2	R2	R2			
6	S7						
7	R4	R4	R4	R4			
8	S2	S10	S3		12		9
9	R5	R5	R5	R5			
10	S2	S6	S3		8	11	
11	R6	R6	R6	R6			
12	R7	R7	R7	R7			

1. S' → S
2. S → aAS
3. S → c
4. A → ba
5. A → SB
6. B → bA
7. B → S

Stack	Input	Action
0	$acbbac\$$	S2
$0a2$	$cbbac\$$	S3
$0a2c3$	$bbac\$$	R3 ($S \rightarrow c$, goto(2,S) = 8)
$0a2S8$	$bbac\$$	S10
$0a2S8b10$	$bac\$$	S6
$0a2S8b10b6$	$ac\$$	S7
$0a2S8b10b6a7$	$c\$$	R4 ($A \rightarrow ba$, goto(10,A) = 11)
$0a2S8b10A11$	$c\$$	R6 ($B \rightarrow bA$, goto(8,B) = 9)
$0a2S8B9$	$c\$$	R5 ($A \rightarrow SB$, goto(2,A) = 4)
$0a2A4$	$c\$$	S3
$0a2A4c3$	$\$$	R3 ($S \rightarrow c$, goto(4,S) = 5)
$0a2A4S5$	$\$$	R2 ($S \rightarrow aAS$, goto(0,S) = 1)
$0S1$	$\$$	R1 ($S' \rightarrow S$), and accept

LR Parsing Example 2 - Parsing Table

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	S5			S4			1	2	3
1		S6				R7 acc			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

1. $E \rightarrow E+T$
2. $E \rightarrow T$
3. $T \rightarrow T*F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow id$
7. $S \rightarrow E$

LR Parsing Example 2(contd.)

Stack	Input	Action
0	$id + id * id\$$	S5
0 id 5	$+id * id\$$	R6 ($F \rightarrow id$, G(0,F) = 3)
0 F 3	$+id * id\$$	R4 ($T \rightarrow F$, G(0,T) = 2)
0 T 2	$+id * id\$$	R2 ($E \rightarrow T$, G(0,E) = 1)
0 E 1	$+id * id\$$	S6
0 E $1 + 6$	$id * id\$$	S5
0 E $1 + 6$ id 5	$*id\$$	R6 ($F \rightarrow id$, G(6,F) = 3)
0 E $1 + 6F3$	$*id\$$	R4 ($T \rightarrow F$, G(6,T) = 9)
0 E $1 + 6T9$	$*id\$$	S7
0 E $1 + 6T9 * 7$	$id\$$	S5
0 E $1 + 6T9 * 7$ id 5	$\$$	R6 ($F \rightarrow id$, G(7,F) = 10)
0 E $1 + 6T9 * 7F10$	$\$$	R3 ($T \rightarrow T * F$, G(6,T) = 9)
0 E $1 + 6T9$	$\$$	R1 ($E \rightarrow E + T$, G(0,E) = 1)
0 E 1	$\$$	R7 ($S \rightarrow E$) and accept

LR Grammars

- Consider a rightmost derivation:
  $S \Rightarrow^*_{rm} \phi B t \Rightarrow_{rm} \phi\beta t$,
  where the production $B \rightarrow \beta$ has been applied
- A grammar is said to be **LR(k)**, if for any given input string, at each step of any rightmost derivation, the handle β can be detected by examining the string $\phi\beta$ and scanning *at most*, first k symbols of the unused input string t

LR Grammars (contd.)

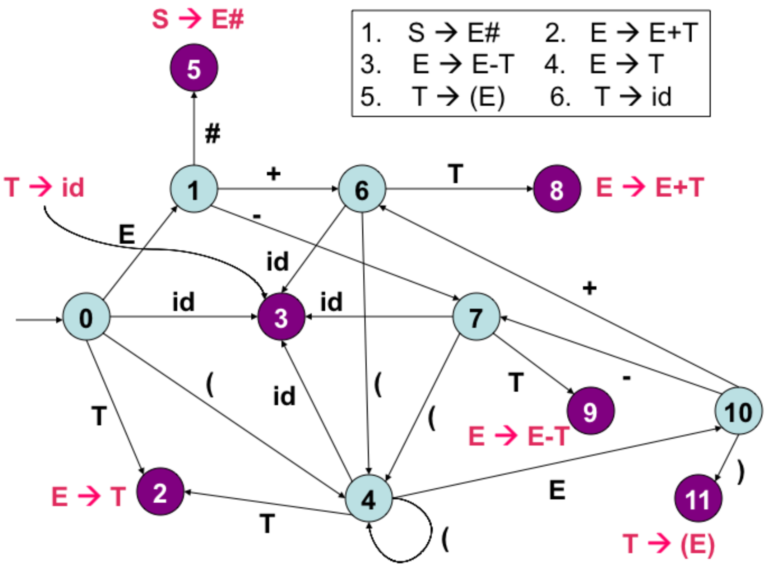
- Example: The grammar, $\{S \rightarrow E,\ E \rightarrow E + E \mid E * E \mid id\}$, is not LR(2)
  - $S \Rightarrow^1 \underline{E} \Rightarrow^2 \underline{E + E} \Rightarrow^3 E + \underline{E * E} \Rightarrow^4 E + E * \underline{id} \Rightarrow^5 E + \underline{id} * id \Rightarrow^6 \underline{id} + id * id$
  - $S \Rightarrow^{1'} \underline{E} \Rightarrow^{2'} \underline{E * E} \Rightarrow^{3'} E * \underline{id} \Rightarrow^{4'} \underline{E + E} * id \Rightarrow^{5'} E + \underline{id} * id \Rightarrow^{6'} \underline{id} + id * id$
  - In the above two derivations, the handle at steps 6 & 6' and at steps 5 & 5', is $E \rightarrow id$, and the position is underlined (with the same lookahead of two symbols, $id+$ and $+id$)
  - However, the handles at step 4 and at step 4' are different ($E \rightarrow id$ and $E \rightarrow E + E$), even though the lookahead of 2 symbols is the same ($*id$), and the stack is also the same ($\phi = E + E$)
  - That means that the handle cannot be determined using the lookahead

LR Grammars (contd.)

- A **viable prefix** of a sentential form $\phi\beta t$, where β denotes the handle, is any prefix of $\phi\beta$. A viable prefix cannot contain symbols to the right of the handle
- Example: $S \rightarrow E\#$, $E \rightarrow E + T \mid E - T \mid T$, $T \rightarrow id \mid (E)$
  $S \Rightarrow E\# \Rightarrow E + T\# \Rightarrow E + (E)\# \Rightarrow E + (T)\#$
  $\Rightarrow E + (id)\#$
  $E,\ E+,\ E + ($, and $E + (id$, are all viable prefixes of the right sentential form $E + (id)\#$
- It is always possible to add appropriate terminal symbols to the end of a viable prefix to get a right-sentential form
- Viable prefixes characterize the prefixes of sentential forms that can occur on the stack of an LR parser

LR Grammars (contd.)

- **Theorem**: The set of all viable prefixes of all the right sentential forms of a grammar is a regular language
- The DFA of this regular language can detect handles during LR parsing
- When this DFA reaches a “reduction state”, the corresponding viable prefix cannot grow further and thus signals a reduction
- This DFA can be constructed by the compiler using the grammar
- All LR parsers have such a DFA incorporated in them
- We construct an augmented grammar for which we construct the DFA
  - If S is the start symbol of G, then G' contains all productions of G and also a new production $S' \rightarrow S$
  - This enables the parser to halt as soon as S' appears on the stack



1. S → E#	2. E → E+T
3. E → E-T	4. E → T
5. T → (E)	6. T → id

Items and *Valid* Items

- A finite set of *items* is associated with each state of DFA
  - An *item* is a marked production of the form $[A \rightarrow \alpha_1.\alpha_2]$, where $A \rightarrow \alpha_1\alpha_2$ is a production and '.' denotes the mark
  - Many items may be associated with a production *e.g.,* the items $[E \rightarrow .E+T]$, $[E \rightarrow E.+T]$, $[E \rightarrow E+.T]$, and $[E \rightarrow E+T.]$ are associated with the production $E \rightarrow E+T$
- An item $[A \rightarrow \alpha_1.\alpha_2]$ is *valid* for some viable prefix $\phi\alpha_1$, iff, there exists some rightmost derivation $S \Rightarrow^* \phi A t \Rightarrow \phi\alpha_1\alpha_2 t$, where $t \in \Sigma^*$
- There may be several items valid for a viable prefix
  - The items $[E \rightarrow E-.T]$, $[T \rightarrow .id]$, and $[T \rightarrow .(E)]$ are all valid for the viable prefix "$E-$" as shown below $S \Rightarrow E\# \Rightarrow E-T\#$, $S \Rightarrow E\# \Rightarrow E-T\# \Rightarrow E-id\#$, $S \Rightarrow E\# \Rightarrow E-T\# \Rightarrow E-(E)\#$

Valid Items and States of LR(0) DFA

- An item indicates how much of a production has already been seen and how much remains to be seen
  - $[E \rightarrow E - .T]$ indicates that we have already seen a string derivable from "$E-$" and that we hope to see next, a string derivable from T
- Each state of an LR(0) DFA contains only those items that are valid for the *same set of viable prefixes*
  - All items in state 7 are valid for the viable prefixes "$E-$" and "($E-$" (and many more)
  - All items in state 4 are valid for the viable prefix "(" (and many more)
  - In fact, the set of all viable prefixes for which the items in a state *s* are valid is the set of strings that can take us from state 0 (initial) to state *s*
- Constructing the LR(0) DFA using sets of items is very simple

Closure of a Set of Items

Itemset closure(I){ /* I is a set of items */
    while (more items can be added to I) {
        for each item $[A \rightarrow \alpha.B\beta] \in I$ {
            /* note that B is a nonterminal and is right after the "." */
            for each production $B \rightarrow \gamma \in G$
                if (item $[B \rightarrow .\gamma] \notin I$) add item $[B \rightarrow .\gamma]$ to I
    }
    return I
}

State 0	State 1	State 7	State 2
S → .E#	S → E.#	E → E-.T	E → T.
E → .E+T	E → E.+T	T → .(E)	
E → .E-T	E → E.-T	T → .id	
E → .T			
T → .(E)			
T → .id			

● indicates closure items

GOTO set computation

Itemset GOTO(I, X){ /* I is a set of items
X is a grammar symbol, a terminal or a nonterminal */
Let $I' = \{[A \rightarrow \alpha X.\beta] \mid [A \rightarrow \alpha.X\beta] \in I\}$;
return (*closure*(I'))
}

State 0
S → .E#
E → .E+T
E → .E-T
E → .T
T → .(E)
T → .id

State 1
S → E.#
E → E.+T
E → E.-T

State 7
E → E-.T
T → .(E)
T → .id

indicates closure items

GOTO(0, E) = 1
GOTO(1, -) = 7

Intuition behind *closure* and *GOTO*

- If an item $[A \rightarrow \alpha.B\delta]$ is in a state (i.e., item set I), then, some time in the future, we expect to see in the input, a string derivable from $B\delta$
  - This implies a string derivable from B as well
  - Therefore, we add an item $[B \rightarrow .\beta]$ corresponding to each production $B \rightarrow \beta$ of B, to the state (i.e., item set I)
- If I is the set of items valid for a viable prefix γ
  - All the items in *closure*(I) are also valid for γ
  - *GOTO*(I, X) is the set items valid for the viable prefix γX
    - If $[A \rightarrow \alpha.B\delta]$ (in item set I) is valid for the viable prefix $\phi\alpha$, and $B \rightarrow \beta$ is a production, we have $S \Rightarrow^* \phi At \Rightarrow \phi\alpha B\delta t \Rightarrow^* \phi\alpha Bxt \Rightarrow \phi\alpha\beta xt$ demonstrating that the item $[B \rightarrow .\beta]$ (in the closure of I) is valid for $\phi\alpha$
    - The above derivation also shows that the item $[A \rightarrow \alpha B.\delta]$ (in GOTO(I, B) is valid for the viable prefix $\phi\alpha B$

Construction of Sets of Canonical LR(0) Items

```
void Set_of_item_sets(G'){ /* G' is the augmented grammar */
   C = {closure({S' → .S})};/* C is a set of item sets */
   while (more item sets can be added to C) {
      for each item set I ∈ C and each grammar symbol X
      /* X is a grammar symbol, a terminal or a nonterminal */
        if ((GOTO(I, X) ≠ ∅) && (GOTO(I, X) ∉ C))
           C = C ∪ GOTO(I, X)
   }
}
```

- Each set in C (above) corresponds to a state of a DFA (LR(0) DFA)
- This is the DFA that recognizes viable prefixes

Construction of an LR(0) Automaton - Example 1

State 0
S → .E#
E → .E+T
E → .E-T
E → .T
T → .(E)
T → .id

State 1
S → E.#
E → E.+T
E → E.-T

State 2
E → T.

State 3
T → id.

State 4
T → (.E)
E → .E+T
E → .E-T
E → .T
T → .(E)
T → .id

State 5
S → E#.

State 6
E → E+.T
T → .(E)
T → .id

State 7
E → E-.T
T → .(E)
T → .id

State 8
E → E+T.

State 9
E → E-T.

State 10
T → (E.)
E → E.+T
E → E.-T

State 11
T → (E).

indicates closure items

indicates kernel items

Shift and Reduce Actions

- If a state contains an item of the form $[A \rightarrow \alpha.]$ ("reduce item"), then a reduction by the production $A \rightarrow \alpha$ is the action in that state
- If there are no "reduce items" in a state, then shift is the appropriate action
- There could be shift-reduce conflicts or reduce-reduce conflicts in a state
  - Both shift and reduce items are present in the same state (S-R conflict), or
  - More than one reduce item is present in a state (R-R conflict)
  - It is normal to have more than one shift item in a state (no shift-shift conflicts are possible)
- If there are no S-R or R-R conflicts in any state of an LR(0) DFA, then the grammar is LR(0), otherwise, it is not LR(0)