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Modeling and Analysis of IEEE 802.15.4 Multi-hop Networks for IoT Applications

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Abstract Multi-hop wireless networks play a crucial role in extending the coverage of monitoring and automation applications in the Internet of Things arena. In this paper, we propose an analytical model for analyzing the performance of IEEE 802.15.4 based multi-hop networks. We accurately model the IEEE 802.15.4 MAC for three different kinds of nodes (leaf, relay and pre-gateway nodes) using 3D Markov chains. Performance of the proposed model is analyzed using reliability, channel congestion and duty cycle as the key performance metrics. The proposed model analyzes the network behavior accurately by achieving less than 5% error when compared with simulation outcomes.

Keywords Markov chains · Multi-hop networks · IoT · WSNs · IEEE 802.15.4-MAC · Monitoring applications

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1 Introduction

Internet of Things (IoT) is rapidly proliferating into multiple domains such as industrial automation, remote monitoring and so forth [1,2,3]. These applications demand stringent requirements such as extended range, reliable data transfer and longer lifetime of the network. IEEE 802.15.4 is one of the widely used IoT communication technologies due to its low power consumption [4]. Many applications such as electrical line monitoring in smart grids [5], fouling detection in ducts system [6] and intelligent load management [7] using IEEE 802.15.4 have emerged recently. IEEE 802.15.4 MAC natively supports single-hop networks and can suffice the needs of automation or monitoring applications within a smaller landscape. However, large industrial deployments require coverage over wider landscapes and require IEEE 802.15.4 MAC based multi-hop networks. One primary challenge wireless monitoring or automation applications face when compared to traditional wired systems is to achieve the needed performance requirements. The lack of analytical models to evaluate accurate performance hinders their adoption in many industrial applications [8,9]. Hence, in this paper, our primary contributions include:

1. Develop mathematical formulation for analyzing the performance of IEEE 802.15.4 MAC for multi-hop networks using 3D Markov chains that can aid in choosing optimal MAC parameters.
2. Propose state models for all the three different kinds of nodes present in the multi-hop networks (leaf, relay and pre-gateway nodes).
3. Validation of proposed mathematical formulation using simulation framework.
4. Performance analysis of IEEE 802.15.4 MAC for multi-hop networks under different network conditions.

Although simulations aid in analyzing the performance of network under different operating conditions, analytical models also aid in choosing optimal MAC parameters for efficient network operation. Conventional approaches use multi-dimensional discrete Markov chains for analyzing the performance of MAC layer and are inspired by the analytical model for IEEE 802.11 MAC [10]. Burratti *et. al.* [11] focused on analyzing the performance of unslotted CSMA/CA by formulating a mathematical model but did not consider the slotted CSMA/CA. Pollin *et. al.* [12] and He *et. al.* [13] later analyzed the performance of slotted CSMA/CA without any retransmissions using a 2D Markov chain. Retransmissions in IEEE 802.15.4 play a significant role in improving the network reliability. Park *et. al.* [14] developed a 3D Markov chain for analyzing the slotted CSMA/CA along with retransmissions and Yung *et. al.* [15] later analyzed the performance of IEEE 802.15.4 CSMA/CA in the presence of superframe. In all these models [11,12,13,14,15], the authors assumed a star network topology where the sensor nodes transmit data to a single gateway. The star network topology will have a smaller coverage and may not fulfill all the needs of IoT applications. Hence, multi-hop communication is a key essential for real time IoT and Cyber Physical Systems (CPS) applications deployed over a large area.

Very few models for analyzing the performance of IEEE 802.15.4 multi-hop networks are present in the literature and are inspired by models proposed for IEEE 802.11 multi-hop networks [16,17,18]. In [16], authors analyzed IEEE 802.11 based multi-hop network with single traffic flow scenario and extended the same for multiple unsaturated flows in [17]. In [18], authors proposed a model for saturated traffic flows in IEEE 802.11 networks. However, due to different channel access mechanisms of IEEE 802.11 and IEEE 802.15.4, these models [16,17,18] cannot be directly applied to IEEE 802.15.4 multi-hop networks. Performance of different IEEE 802.15.4 multi-hop network topologies with heterogeneous traffic and hidden node terminal problems is discussed in [19]. Even though this model captures the multi-hop behavior of a particular topology, due to its routing layer dependencies, analyzing a larger network involves significant complexity. Reliable and timely communication from sensors to sink is discussed in [20] using anycast multi-hop routing between relay clusters. Reliability, delay, and energy consumption of overall network are analyzed with cumulative exponential wake-up rate, traffic rate and the number of relay clusters. Model in [20] also proposed different states like sleep, idle-listen, active and CSMA/CA for relay nodes but the analytical model has fixed backoff window slots and lacks many network parameters. An analytical model for evaluating the performance of IEEE 802.15.4 multi-hop networks based on beacon-less CSMA/CA is proposed in [22]. In [22], authors analyzed the performance of multi-hop tree network with sensors and relay nodes (relay node can be sensors as well) in the absence of beacons.

To develop a robust analytical model which can guide user in deciding MAC parameters to achieve desired reliability lies as the key motivation for this work. In this work an IEEE 802.15.4 based multi-hop network with modified MAC layer is proposed to address network scalability using anycast multi-hop clustered networks which avoids the dependency on routing. Sleep, Idle-Listen, Active-Tx and CSMA/CA states are introduced into the leaf, relay and gateway nodes to attain energy efficiency without compromising on the end-to-end reliability, delay and throughput.

The remainder of this manuscript is organized as follows. Section II presents the holistic view of the complete multi-hop network architecture. Modeling and mathematical formulation of individual node behavior is discussed in section III. Validation of the proposed model using simulations and the performance is discussed in Section IV. Finally, Section V concludes the paper by discussing the future scope of this work.

2 Holistic View of the IEEE 802.15.4 Multi Hop Network Architecture

Typically in monitoring applications such as environmental, habitat and industrial, the area of coverage should be made as large as possible. This can only

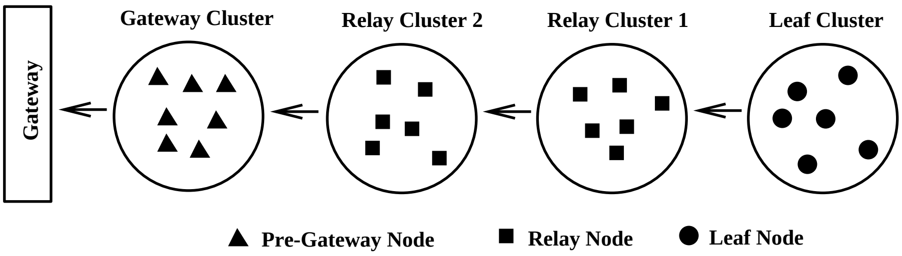


Fig. 1 Network topology of multi-hop WSN

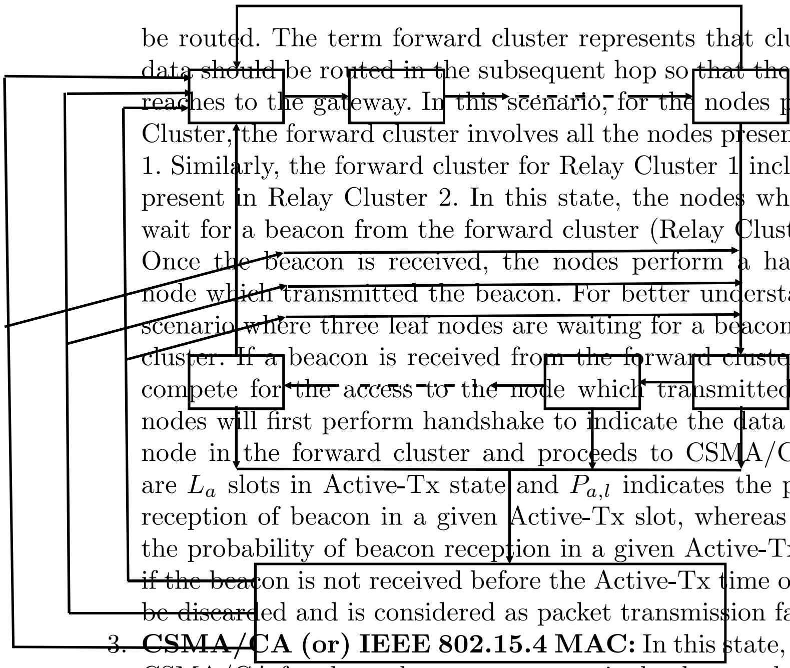
be achieved with the usage of multi-hop networks. A typical and commonly used multi-hop wireless sensor network topology for monitoring applications is shown in Fig. 1. In Fig. 1, the data is generated at the leaf cluster where the sensor or leaf nodes are present. This generated data from sensors at the leaf nodes should be transmitted to the gateway with the help of intermediate relays. The primary functionality of relay nodes includes the data transmission from leaf nodes to gateway within a finite number of hops thereby guaranteeing the reliability of the sensed data. In few applications, the intermittent nodes may also have capabilities to generate data in addition to relaying the data being generated at the leaf nodes. Whereas in this study we are limiting the purpose of the intermittent nodes only to route the data generated at leaf cluster to the gateway. Following sections briefly, describe the life cycle of nodes present in each of the cluster.

2.1 Leaf Cluster

The leaf cluster acts as the primary source for data generation. Nodes present in the leaf cluster, also known as leaf nodes sense the physical parameters of interest and transmit using IEEE 802.15.4 technology. The behavior of a leaf node is modeled using 3D Markov chain as shown in Fig. 2. In general, the leaf node spends its life cycle in 3 different states namely Sleep, Active-Tx and CSMA/CA. The behavior of a leaf node in each of these states is provided below.

1. **Sleep state (S_0 to S_{h-1}):** In the sleep state, the node is put to sleep for achieving energy conservation. Ideally, the power consumption in this state is zero, but practically it will be as low as possible and includes only the power consumed by the wake-up controller of the sensing node. The states S_{h-1} to S_0 indicates the total h slots present in the sleep state. When the node is in its last slot of Sleep state, the node generates a packet with a probability of η_l. If a packet is generated, the node immediately transits to Active-Tx state, else the node transits back to the sleep state and sleeps for the next h slots then repeats the same process.
2. **Active-Tx (A_0 to A_{L_a-1}):** This state primarily supports for selection of a node in the forward cluster, to which the generated packet has to

Fig. 2 Markov Chain based model for leaf node



be routed. The term forward cluster represents that cluster to which the data should be routed in the subsequent hop so that the packet ultimately reaches to the gateway. In this scenario, for the nodes present in the Leaf Cluster, the forward cluster involves all the nodes present in Relay Cluster 1. Similarly, the forward cluster for Relay Cluster 1 includes all the nodes present in Relay Cluster 2. In this state, the nodes which have data will wait for a beacon from the forward cluster (Relay Cluster 1 in this case). Once the beacon is received, the nodes perform a handshake with the node which transmitted the beacon. For better understanding, consider a scenario where three leaf nodes are waiting for a beacon from the forward cluster. If a beacon is received from the forward cluster, the three nodes compete for the access to the node which transmitted the beacon. The nodes will first perform handshake to indicate the data availability to the node in the forward cluster and proceeds to CSMA/CA process. There are L_a slots in Active-Tx state and $P_{a,l}$ indicates the probability for not reception of beacon in a given Active-Tx slot, whereas $1 - P_{a,l}$ indicates the probability of beacon reception in a given Active-Tx slot. In any case if the beacon is not received before the Active-Tx time out, the packet will be discarded and is considered as packet transmission failure.

3. **CSMA/CA (or) IEEE 802.15.4 MAC:** In this state, the node performs CSMA/CA for channel access to transmit the data packets generated. The detailed modeling of IEEE 802.15.4 MAC is shown in Fig. 3 with backoff stages (m), backoff counter (k) and collision retries (n) as the three dimensions. In the CSMA/CA, the node waits for a random backoff period and senses the channel during CCA1 and CCA2 slots ($b_{0,k,j}$ and $b_{-1,k,j}$). If the channel is found free during CCA1 and CCA2, the node proceeds with data transmission else the node generates an exponentially increased random backoff and performs the same process until the maximum number of backoff stages are exceeded. If the maximum number of back off stages are exceeded the packet will then be discarded. In the case where node gains channel access, the node transmits the data and P_c indicates the probability of occurrence of a collision in the channel. If any collisions occur, the node will retry to transmit the packet again until a maximum number of retries are exceeded. In Fig. 3, α and β indicate the probability of the channel being busy in CCA1 and CCA2 slots respectively, whereas n indicates the maximum retries allowed.

2.2 Relay Cluster

Reception of data from the backward cluster and forwarding it to the forward cluster is the primary goal of relay nodes present in the relay cluster. The life cycle of the relay node can be modeled as shown in Fig. 4. Fig. 4 shows

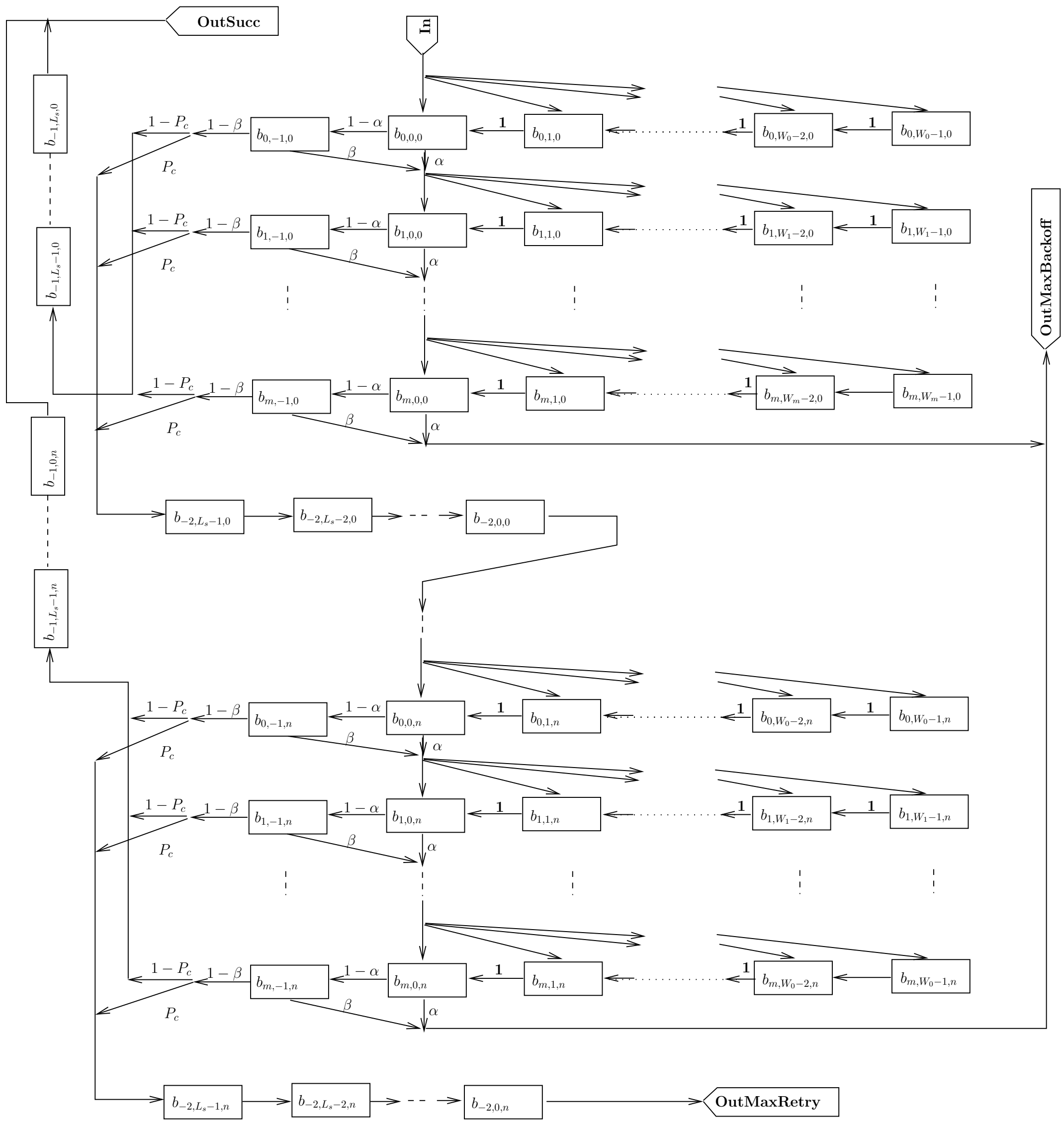


Fig. 3 3D Markov Chain model for IEEE 802.15.4 MAC

the Markov chain model for relay node functionality. The functionality can be classified into four states namely Sleep, Idle-Listen, Active-Tx and CSMA/CA. The primary functionality of the relay nodes in the states Sleep, Active-Tx and CSMA/CA is similar to the functionality of leaf nodes. Relay nodes in Idle-Listen (I_0 to I_{L_i-1}) state waits for L_i slots for a packet from the backward cluster. I_0 to I_{L_i-1} indicate the Idle-Listen slots and $P_{i,r}$ is the probability of not receiving a packet in a given Idle-Listen slot. If a packet is received within

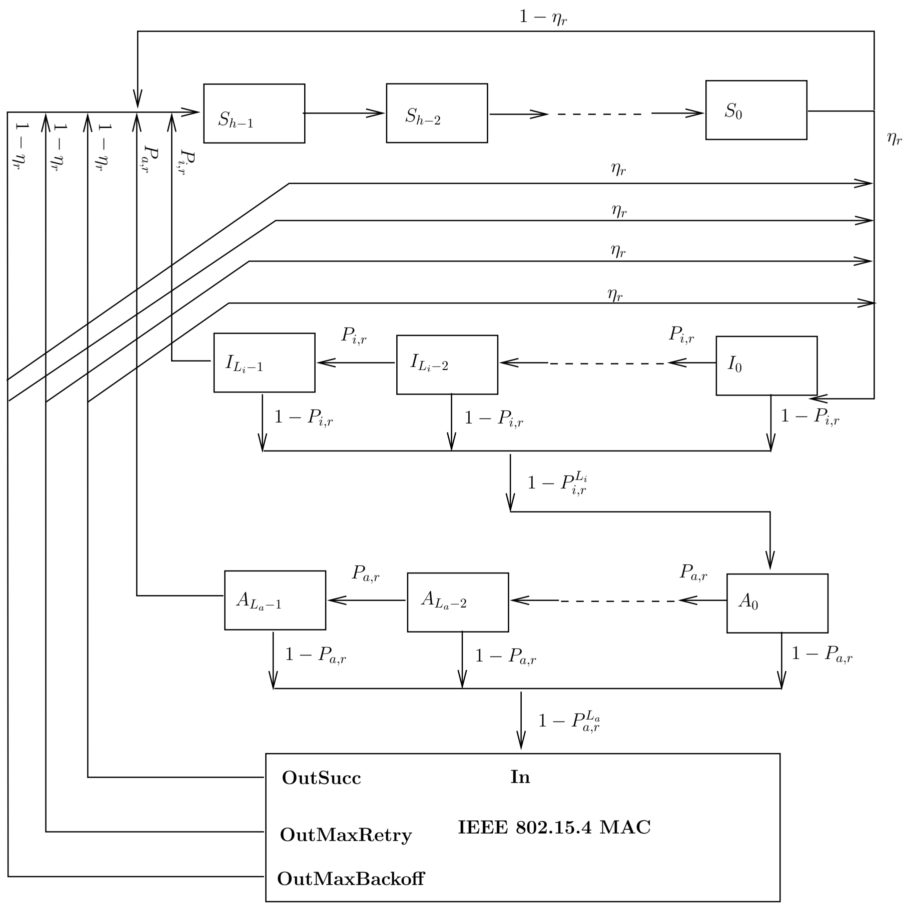


Fig. 4 Markov Chain model for relay node

L_i slots, the relay node transits to Active-Tx state and forwards the packet to forward cluster.

2.3 Gateway Cluster

The nodes present in Gateway cluster are termed as pre-gateway nodes. These nodes will directly communicate the data aggregated from relay and leaf clusters. The nodes in this cluster need not have to wait for beacons from the gateway. A gateway in this scenario is a node which aggregates the data from all the leaf nodes and spends its complete life cycle in ON state. The pre-gateway nodes can always transmit the data whenever the channel is found to be free. Hence the life cycle of a pre-gateway node consists of states namely

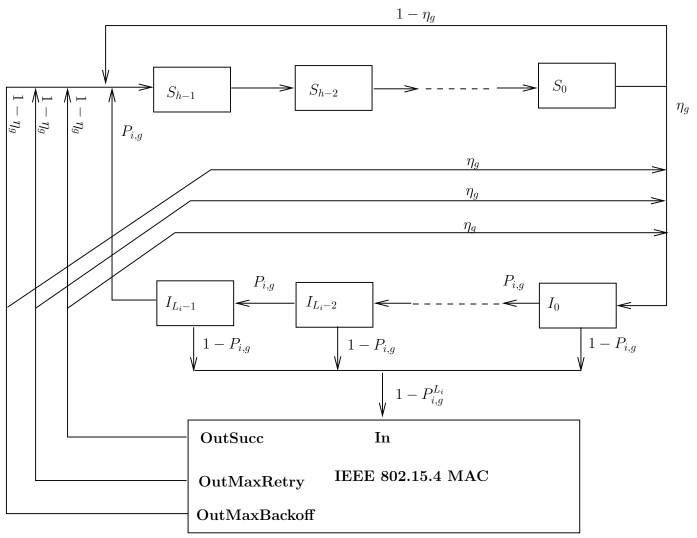


Fig. 5 Markov Chain model for pre-gateway node

Sleep, Idle-Listen, CSMA/CA as shown in Fig. 5. Also, the behavior of nodes in these three states is similar to that of discussed in earlier sections.

3 Analytical Modeling

The analytical modeling of the network considered is performed using the above described 3D Markov chains for leaf, relay and pre-gateway nodes. Following sub sections describe the assumptions considered in this study and approach adhered for modeling.

3.1 Assumptions

The following are the assumptions considered in this study

- Effect of handshaking mechanism is assumed to be negligible.
- Nodes receive and transmit in different frequencies. This isolates channel contention within the cluster and also improves the performance.

To elaborate the second assumption, consider the channel between Leaf Cluster and Relay Cluster 1 to be C1 and between Relay Cluster 1 to Relay Cluster 2 be C2. Assuming the leaf nodes and relay nodes transmit the same frequency, for a relay node to transmit there should not be any on going transmissions from both the Relay Cluster 1 and Leaf Cluster. With this assumption, the relay nodes and leaf nodes transmit at different frequencies and channels operate independently.

Table 1 Description of notations used for analytical modeling. (*Note:* Below notations represent the probability for nodes in Lear Cluster. For all the other clusters the probabilities are represented using additional term in superscript. Eg: $b_{0,0,0,r}$ represent the probability of a relay node of Relay Cluster r residing in state $(0, 0, 0)$)

Notation	Description
$b_{0,0,0}$	Probability of a node residing in state $0, 0, 0$
P_{A_0}	Probability of a node residing in A_0 state of Active-Tx state
P_{I_0}	Probability of a node residing in I_0 state of Idle-Listen state
η	Probability of packet availability for leaf node
$P_{a,l}$	Probability of non-reception of beacon in Active-Tx state
L_a	Length of Active-Tx state
L_i	Length of Idle-Listen state
m	Maximum backoff stages in CSMA/CA
n	Maximum no. of retransmissions in CSMA/CA
P_c	Probability of collision
α	CCA1 failure probability
β	CCA2 failure probability
R	Reliability of the node
L_S	Length of successful packet transmission
L_C	Length of packet collision
N	No. of nodes in the network
h	Length of Sleep state

3.2 Approach for modeling

To accurately model the network behavior we started with modeling the behavior of leaf node by providing reliability as the constraint to be met. The output of this is used to tune the wake-up rate of the relay node thus guaranteeing enough number of beacons are received by Leaf Cluster to achieve the required reliability. The same approach will then be used to tune rest of the nodes which include relay and pre-gateway nodes in the forward clusters. After accurately modeling the nodes, the duty cycle modeling for all the nodes will be performed. Table 1 summarizes all the important notations used for modeling.

3.3 Analytical model for leaf node

We model the behavior of leaf node in two stages. The first stage involves derivation of transition and state probabilities followed by the derivation of necessary parameters such as channel congestion, reliability etc. Fig. 2 depicts the 3D Markov chain model for a leaf node. Equation (1), gives the probability of a node residing in the first slot of Active state at any given random time slot expressed in terms of P_{S_0} and $b_{0,0,0}$. Where as (2) provides the probability of a node residing in $b_{0,0,0}$. Upon using (1) and (2), we can rewrite $b_{0,0,0}$ as shown in (3) and express $b_{0,0,0}$ in terms of P_{S_0} which is shown in (4).

$$P_{A_0} = P_{S_0}\eta + b_{0,0,0}\eta \tag{1}$$

$$b_{0,0,0} = P_{A_0}(1 - P_{a,l}^{L_a}) \tag{2}$$

$$b_{0,0,0} = (P_{S_0}\eta + b_{0,0,0}\eta)(1 - P_{a,l}^{L_a}) \tag{3}$$

$$b_{0,0,0} = \frac{P_{S_0}\eta(1 - P_{a,l}^{L_a})}{1 - \eta(1 - P_{a,l}^{L_a})} \tag{4}$$

Now we make use of normalization property of Markov chains to derive the additional state probability information. By utilizing the property of normalization, which defines that sum of probabilities of all the states in which a node can spend in its life time equals to 1. P_S and P_A indicate the probability of the leaf node residing in Sleep and Active-Tx states at any given random time slot, whereas P_{CSMA} denotes the probability of a node residing in CSMA/CA state. Now we derive individual state probabilities and make use of the normalization property given in (5) to calculate reliability, and duty cycle of the leaf node.

$$P_S + P_A + P_{CSMA} = 1 \tag{5}$$

Equation (6) indicates the total Sleep state probability in terms of P_{S_0} while (7), (8) and (9) expresses Active-Tx state probability in terms of P_{S_0}

$$P_S = hP_{S_0} \tag{6}$$

$$P_A = P_{A_0} + P_{A_1} + ... + P_{A_{L_a-1}} \tag{7}$$

$$P_A = P_{A_0}(1 + P_{a,l} + P_{a,l}^2 + ... + P_{a,l}^{L_a-1}) \tag{8}$$

$$P_A = \frac{\eta P_{S_0}(1 - P_{a,l}^{L_a})}{1 - P_{a,l}} \tag{9}$$

The probability of a node residing in CSMA/CA state expressed in terms of $b_{0,0,0}$ is given by (10). In equation (11), to improve the readability, the probability P_{CSMA} is represented using a constant K, which isolates $b_{0,0,0}$ with rest of the expression.

$$\begin{aligned} P_{CSMA} =& \frac{b_{0,0,0}}{2}\left(\frac{1-(2x)^{(m+1)}}{1-2x}W_0 + \frac{(1-x^{(m+1)})}{1-x}\right)\frac{1-y^{(n+1)}}{1-y} \\ &+ (1-\alpha)\frac{1-x^{(m+1)}}{1-x}\frac{1-y(n+1)}{1-y}b_{0,0,0} \\ &+ \left(L_S(1-P_c) + L_C P_c)(1-x^{(m+1)})\frac{1-y^{(n+1)}}{1-y}b_{0,0,0}\right) \end{aligned} \tag{10}$$

$$P_{CSMA} = Kb_{0,0,0} \tag{11}$$

Now by substituting in (5), we can derive the probability of a node residing in Sleep state as shown in (13).

$$P_{S_0}\left[h+\eta\frac{1-P_{a,l}^{La}}{1-P_{a,l}}\right]+b_{0,0,0}\left[\eta\frac{1-P_{a,l}^{La}}{1-P_{a,l}}+K\right]=1 \tag{12}$$

$$P_{S_0}=\frac{1}{h+\eta\frac{(1-P_{a,l}^{La})}{1-P_{a,l}}+\frac{\eta(1-P_{a,l}^{La})}{1-\eta(1-P_{a,l}^{La})}\left(\eta\frac{1-P_{a,l}^{La}}{1-P_{a,l}}+K\right)} \tag{13}$$

In [14], the authors have modeled the IEEE 802.15.4 MAC and analyzed the performance with various MAC parameters. We make use of the same analysis to derive the probabilities of channel congestion etc., for modeling the CSMA/CA proposed in Fig. 3. Equation (14) gives the probability τ that a node attempts CCA1 in a randomly chosen time slot. The term P_c shown in (15) is the probability that at least one of the $N-1$ remaining nodes transmit in the same time slot, also known as the probability of collision.

$$\tau=\frac{1-x^{(m+1)}}{1-x}\frac{1-y^{(n+1)}}{1-y}b_{0,0,0} \tag{14}$$

$$P_c=1-(1-\tau)^{(N-1)} \tag{15}$$

In equation (14), the parameters x and y are given by (16) and (17). Similarly the busy probabilities CCA1 and CCA2 which are α and β are given by (18) and (19).

$$x=\alpha+(1-\alpha)\beta \tag{16}$$

$$y=P_c(1-x^{m+1}) \tag{17}$$

$$\begin{aligned}\alpha =&L_p(1-(1-\tau)^{(N-1)})(1-\alpha)(1-\beta)+\\ &L_{ack}\frac{N\tau(1-\tau)^{(N-1)}}{1-(1-\tau)^{(N-1)}}(1-\tau)^{(N-1)})(1-\alpha)(1-\beta)\end{aligned} \tag{18}$$

$$\beta=\frac{1-(1-\tau)^{N-1}+N\tau(1-\tau)^{N-1}}{2-(1-\tau)^{N}+N\tau(1-\tau)^{N-1}} \tag{19}$$

Now we derive the probability of successful transmission (or) reliability by calculating the probability that a packet is discarded due to timeout in Active-Tx, channel access failure and exceeding retry limits. The reliability of a leaf node is given by equation (20).

$$R=1-P_{a,l}^{L_a}-(1-P_{a,l}^{L_a})\left\{\frac{x^{m+1}(1-y^{n+1})}{1-y}+y^{n+1}\right\} \tag{20}$$

Using the derived reliability, we calculate the probability of non reception of a beacon for a given node in a random time slot as shown in equation (21).

$$P_{a,l} = \left[1 - R - (1 - P_{a,l}^{L_a})\left\{\frac{x^{m+1}(1-y^{n+1})}{1-y} + y^{n+1}\right\}\right]^{\frac{1}{L_a}} \tag{21}$$

Now assuming each of the relay node transmits beacon with a probability of $P_{bp,r}$, we can represent $P_{a,l}$ as shown in (22).

$$P_{a,l} = 1 - \frac{NP_{bp,r}}{1 + (N-1)P_A} \tag{22}$$

Using equation (22), the probability with which the relay node has to transmit beacon to the Leaf Cluster in any given random time slot is given by (23).

$$P_{bp,r} = \frac{1 + (N-1)P_A(1 - P_{a,l})}{N} \tag{23}$$

We will make use of the probability $P_{bp,r}$ calculated using (23) when modeling the relay node to determine its wake-up rate.

3.4 Analytical model for relay node

Similar to the modeling of the leaf node, we will put a constraint on the reliability the relay node needs to deliver. Using this constraint, we derive the probability with which the beacons have to be received from the forward cluster. Considering Fig. 4, we can derive the probability of a relay node from relay cluster r residing in S_0 as shown in (24). The suffix r in (24) signifies the node is from cluster r (relay cluster). The similar symbolic convention will be adhered for all notations described in Table 1 for clearly discriminating the probabilities across the clusters.

$$P_{S_0,r} = (1-\eta_r)P_{S_0,r} + (1-\eta_r)b_{0,0,0,r} + P_{a,r}^{L_{a,r}}P_{A_0,r}(1-\eta_r) + P_{i,r}^{L_{i,r}}P_{I_0,r} \tag{24}$$

Where $P_{A_0,r}$ indicates the probability of a relay node staying in A_0 state at any given random time slot, $P_{I_0,r}$ indicates the probability of a relay node staying in I_0. $P_{a,r}$ and $P_{i,r}$ indicate the probability for non-reception of beacon and packet in Active and Idle slots respectively. $P_{A_0,r}$ can be expressed in terms of $P_{I_0,r}$ as shown in (25).

$$P_{A_0,r} = (1 - P_{i,r}^{L_{i,r}})P_{I_0,r} \tag{25}$$

Now, the probability $P_{I_0,r}$ can be expressed as shown in (26). Using (25) and (26), the probability $P_{I_0,r}$ can be expressed as shown in (27). By making use of (27), the $b_{0,0,0,r}$ (probability of a relay node residing in *(0,0,0)*) can be expressed in terms of $P_{S_0,r}$ as shown in equation (28) and (29) where A indicates the constant of proportionality used for improving readability.

$$P_{I_0,r} = \eta_r P_{S_0,r} + \eta_r b_{0,0,0,r} + \eta_r P_{a,r}^{L_{a,r}}P_{A_0,r} \tag{26}$$

$$P_{I_0,r} = \frac{\eta_r P_{S_0,r} + \eta_r b_{0,0,0,r}}{1 - \eta_r P_{a,r}^{L_{a,r}}(1 - +P_{i,r}^{L_{i,r}})} \tag{27}$$

$$b_{0,0,0,r} = P_{S_0,r}\frac{\eta_r - \eta_r P_{a,r}^{L_{a,r}}(1 - P_{i,r}^{L_{i,r}}) - \eta_r P_{i,r}^{L_{i,r}}}{1 - \eta_r} \tag{28}$$

$$b_{0,0,0,r} = AP_{S_0,r} \tag{29}$$

Using the normalization criteria shown in (30), probability of a relay node residing in S_0 state is shown in (32). The probability P_{CSMA} remains same as that of the leaf node.

$$P_S + P_A + P_I + P_{CSMA} = 1 \tag{30}$$

$$P_{S_0,r}h_r + \frac{1 - P_{i,r}^{L_{i,r}}}{1 - P_{i,r}}P_{I_0,r} + \frac{1 - P_{a,r}^{L_{a,r}}}{1 - P_{a,r}}P_{A_0,r} + Kb_{0,0,0,r} = 1 \tag{31}$$

$$\begin{aligned} P_{S_0,r} = \Bigg[& h_r + \frac{\eta_r(1 - P_{i,r}^{L_{i,r}})}{(1 - P_{i,r})(1 - \eta_r P_{a,r}^{L_{a,r}}(1 - P_{i,r}^{L_{i,r}}))} + \\ & \frac{\eta_r(1 - P_{i,r}^{L_{i,r}})(1 - P_{a,r}^{L_{a,r}})}{(1 - P_{a,r})(1 - \eta_r P_{a,r}^{L_{a,r}}(1 - P_{i,r}^{L_{i,r}}))} + \\ & A\Big(K + \frac{\eta_r(1 - P_{i,r}^{L_{i,r}})}{(1 - P_{i,r})(1 - \eta_r P_{a,r}^{L_{a,r}}(1 - P_{i,r}^{L_{i,r}}))} + \\ & \frac{\eta_r(1 - P_{i,r}^{L_{i,r}})(1 - P_{a,r}^{L_{a,r}})}{(1 - P_{a,r})(1 - \eta_r P_{a,r}^{L_{a,r}}(1 - P_{i,r}^{L_{i,r}}))}\Big)\Bigg]^{-1} \end{aligned} \tag{32}$$

The reliability of a relay node can be expressed as shown in (33). The expressions for α, β, x and y remains same as that of the leaf node.

$$R = 1 - P_{a,r}^{L_{a,r}} - (1 - P_{a,r}^{L_{a,r}})\left\{\frac{x^{m+1}(1 - y^{n+1})}{1 - y} + y^{n+1}\right\} \tag{33}$$

Using the reliability derived, the probability of non reception of a beacon for a given relay node is

$$P_{a,r} = \left[1 - R - (1 - P_{a,l}^{L_a})\left\{\frac{x^{m+1}(1 - y^{n+1})}{1 - y} + y^{n+1}\right\}\right]^{\frac{1}{L_a}} \tag{34}$$

By assuming each of the relay node in the forward cluster (can be Relay Cluster 2 for Relay Cluster 1 and Gateway Cluster to Relay Cluster 2 in this case) transmits beacon with a probability of $P_{bp,r}$, we can represent $P_{a,r}$ as

$$P_{a,r} = 1 - \frac{NP_{bp,r}}{1 + (N - 1)P_A} \tag{35}$$

In (35), P_A indicates the probability of a relay node staying in Active-Tx state. Using (35), we can calculate the probability that a relay node needs to transmit a beacon to Leaf Cluster in any given random time slot as shown in (36).

$$P_{bp,r} = \frac{1 + (N-1)P_A(1 - P_{a,r})}{N} \tag{36}$$

We will make use of the probability $P_{bp,r}$ calculated using (36) when modeling the relay node to determine its wake-up rate. In the analysis of leaf node, we derived the probability at which a relay node in Relay Cluster 1 has to transmit a beacon in a given time slot to be $P_{bp,r}$. Using this, the wake-up rate with which the relay node has to function can be given as

$$\eta_r = \frac{P_{bp,r}}{P_{S_0,r} + b_{0,0,0,r} + \eta_r P_{a,r}^{L_{a,r}} P_{A_0,r}} \tag{37}$$

The relay model analysis performed here can be extended in the same way for the Relay Cluster 2 with minimal change of considerations such as $P_{bp,r}$ should be derived using the requirement of Relay Cluster 1.

3.5 Analytical model for pre-gateway node

For modeling the behavior of a pre-gateway node, we will make use of the Markov chain model shown in Fig. 5. As shown in Fig. 5, the pre-gateway node does not have Active-Tx state. Hence, the probability of the node residing in S_0 at any given random time can be expressed as shown in (38). $P_{I_0,g}$ as shown in (39), indicates the probability of a node residing in I_0 of the Idle-Listen state. By making use of (38) and (39), we derive the probability $b_{0,0,0,g}$ as shown in (40).

$$P_{S_0,g} = (1-\eta_g)P_{S_0,g} + (1-\eta_g)b_{0,0,0,g} + P_{i,g}^{L_{i,g}} P_{I_0,g} \tag{38}$$

$$P_{I_0,g} = \eta_g P_{S_0,g} + \eta_g b_{0,0,0,g} \tag{39}$$

$$b_{0,0,0,g} = P_{S_0,g} \frac{\eta_g(1 - P_{i,g}^{L_{i,g}})}{1 - \eta_g + \eta_g P_{i,g}^{L_{i,g}}} \tag{40}$$

Now using the normalization criterion of Markov chains shown in (41), we derive $P_{S_0,g}$ as shown in (43).

$$P_S + P_I + P_{CSMA} = 1 \tag{41}$$

$$P_{S_0,g} h_g + \frac{1 - P_{i,g}^{L_{i,g}}}{1 - P_{i,g}} (\eta_g P_{S_0,g} + \eta_g b_{0,0,0,g}) + K b_{0,0,0,g} = 1 \tag{42}$$

$$P_{S_0,g} = \left[h_g + \eta_g \frac{1 - P_{i,g}^{L_{i,g}}}{1 - P_{i,g}} + A \left(K + \eta_g \frac{1 - P_{i,g}^{L_{i,g}}}{1 - P_{i,g}} \right) \right]^{-1} \tag{43}$$

The reliability can now be derived in similar way as that of shown in leaf node modeling and is

$$R = 1 - \left\{ \frac{x^{m+1}(1 - y^{n+1})}{1 - y} + y^{n+1} \right\} \tag{44}$$

By making use of $P_{bp,r}$ calculated in the previous section of relay node modeling, the wake-up rate of the node in pre-gateway cluster can be given as

$$\eta_r = \frac{P_{bp,r}}{P_{S_0,r} + b_{0,0,0,r}} \tag{45}$$

3.6 Duty cycle modeling

The duty cycle of the nodes which determine the fraction of the time a node spends other than in Sleep state can be calculated using the equations (46), (47) and (48) respectively for leaf, relay and pre-gateway node

$$DC_l = 1 - hP_{S_0} \tag{46}$$

$$DC_r = 1 - h_r P_{S_0,r} \tag{47}$$

$$DC_g = 1 - h_g P_{S_0,g} \tag{48}$$

4 Performance Analysis

For the performance analysis we considered the network with one leaf cluster (l), two relay clusters (r1, r2) and a gateway cluster (g). The proposed model is validated by comparing the analytical results with the simulation results. Fig. 6, depicts the approach followed for the performance analysis. Following steps detail the approach.

- Define the reliability of each node in every cluster
- Derive the wake-up rate of each node required in every cluster from the developed analytical model
- Use thus found wake-up rates in the simulation model
- Compare the MAC layer performance using parameters α, β, reliability and duty cycle of nodes in individual clusters.

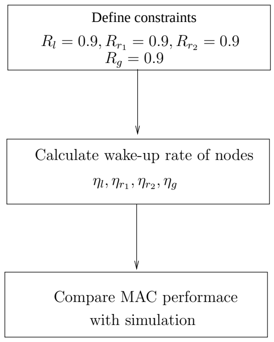


Fig. 6 Approach for performance analysis

Authors in [23], developed a simplistic Monte-Carlo simulation model for analyzing the MAC layer performance of leaf node under ideal channel conditions. We made use of the simulation model proposed in [23] and extended it to support the multi-hop communications. Our primary assumptions in the developed simulation model include:

- The traffic generated due to periodic beacon communication is assumed to be negligible
- Effect of handshaking is negligible
- Nodes receive and transmit in different frequencies.

Unless and until specified, the parameters assumed for the nodes in every cluster are as follows $N = 10,\ L_i = 300,\ L_a = 150,\ \eta = 0.03,\ R = 0.9$. Fig. 7 shows the variation of wake-up rate of a relay node (η_{r_1}) in relay cluster 1 with the variation in traffic generation at leaf node (η_l). As the traffic generation increases, to maintain a constant reliability, the number of beacons that need to be received from the forward cluster should also be increased. The same behavior can also be observed from Fig. 7, where the wake-up rate increases with the traffic generation. Similarly, as the number of packets for transmission increases, the amount of congestion in the network also increases which can be observed from Fig. 8. Fig. 9 and Fig. 10 plots the variation in wake-up rate and channel congestion in every cluster respectively with variation in the traffic generation rate, respectively. From Fig. 9, one can observe the increase in wake-up rate of nodes with increase in traffic generation rate at the leaf node. With increase in number of packets the node should wake up more often to ensure a required amount of beacons are being sent to achieve the user specified reliability. Also, as the traffic increases, channel congestion also increases and more number of failures will be observed in CCA1, which leads to increase in α in every cluster. The same observation can be inferred from the Fig. 10. With constant traffic generation rate of $\eta_l = 0.03$, the increase in number of nodes present in the relay cluster results in a decrease of wake-up rate per node in relay cluster 1. Since, the leaf node needs a specific amount of beacons to be received from relay cluster to achieve the defined reliability of

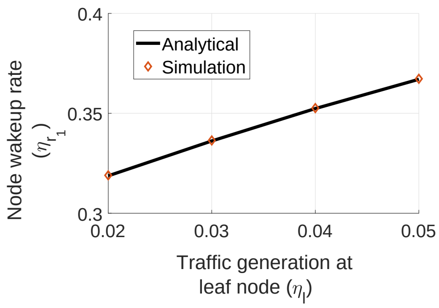


Fig. 7 Analysis of wake-up rate of a relay node in relay cluster 1 with variation of traffic generation at leaf node

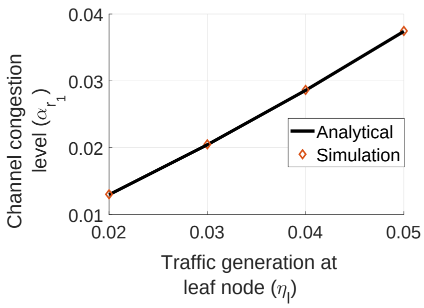


Fig. 8 Analysis of congestion in relay cluster 1 with variation of traffic generation at leaf node

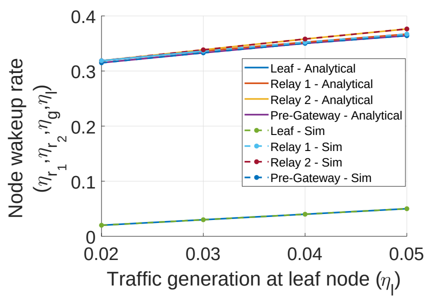


Fig. 9 Analysis of wake-up rate of nodes in every cluster with variation of traffic generation at leaf node

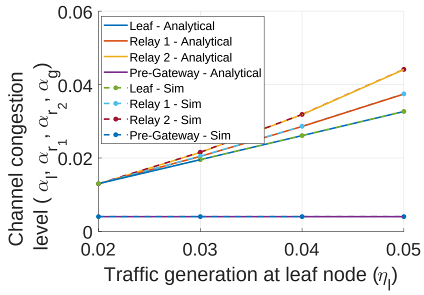


Fig. 10 Analysis of congestion in every cluster with variation of traffic generation at leaf node

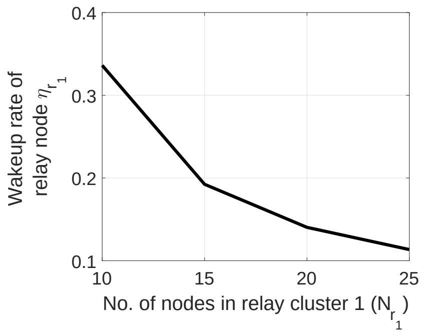


Fig. 11 Analysis of wake-up rate of relay node in relay cluster 1 with variation of number of nodes in relay cluster 1 ($R_l = R_{r_1} = R_{r_2} = R_g = 0.9, \eta_l = 0.03, N_l = N_{r_2} = N_g = 10$)

0.9 instead of total nodes present in the relay cluster, as we increase number of nodes, the wake-up rate per node decreases. Hence, we can observe the decrease in wake-up rate of relay nodes as shown in Fig. 11. Fig. 12 shows the variation of channel congestion with a change in the total number of sleep slots in every cluster. As the length of sleep slots increase, the leaf node spends most of its time in sleep mode which in turn reduces the amount of data generated at the leaf nodes leading to a reduction of congestion in all the clusters. Since the length of the sleep states increase, the probability of node spending in Active-Tx state decreases, hence to achieve the reliability the beacon rate needs to be increased. Hence, from Fig. 13, which shows the variation in wake-up rate with variation in length of Sleep state, we can see the increase in the wake-up rate of the nodes. Fig. 14 and Fig. 15 plots the variation of duty cycle of a relay node present in Relay Cluster 1 with variation in traffic generation and number of nodes per cluster respectively. From Fig. 14, one can observe the increase in duty cycle of relay node with increase in the traffic generation. As the traffic generation increases, the node spends most of its life cycle in active mode to achieve the user specified reliability. Also, as the number of nodes

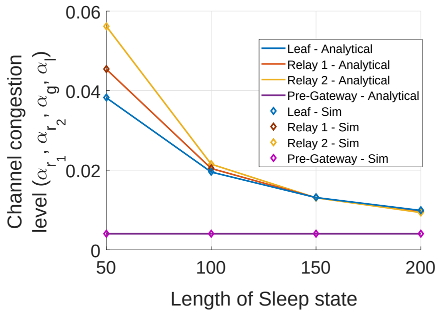


Fig. 12 Analysis of channel congestion in each cluster with variation in h

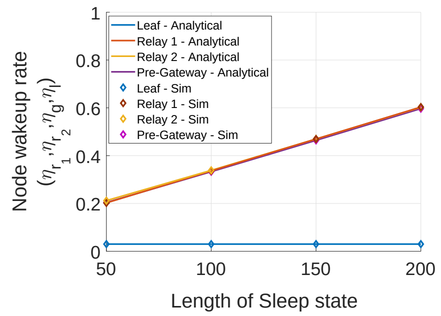


Fig. 13 Analysis of wake-up rate of each node in every cluster with variation in h

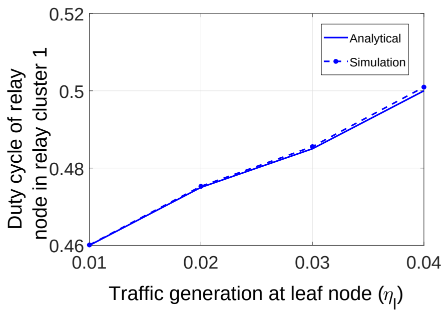


Fig. 14 Analysis of duty cycle of relay node in relay cluster 1 with variation in traffic generation

per cluster increases, the wake-up rate of the relay node decreases which in turn reduces the duty cycle. The same behavior can be observed from the Fig. 15. From the above performance analysis, one can observe that the proposed analytical model accurately analyzes the performance of IEEE 802.5.4 multi-hop networks accurately by achieving less than 5% error when validated using

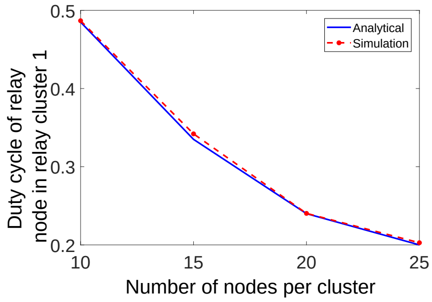


Fig. 15 Analysis of wake-up rate of a node in relay cluster 1 with variation in number of nodes per cluster

simulation outcomes. Although the model considered here has only four hops (Leaf to Relay 1, Relay Cluster 1 to Relay Cluster 2, Relay Cluster 2 to Gateway Cluster and Gateway Cluster to Gateway), it can also be extended to any number of hops with the usage of same relay node model.

5 Conclusion

In this paper, we proposed a novel analytical model for IEEE 802.15.4 multi-hop networks using 3D Markov chains. This study formulates the mathematical model for the complete network by considering the user specified reliability constraint and decides the wake-up rate for nodes present in various clusters. From the performance analysis it is observed that the proposed analytical model and simulation outcomes are in good agreement by achieving less 5% error. We are convinced that this study can greatly aid in driving the future research in modeling of multi-hop networks for IoT applications. Our future scope is to develop a real time test bed for the proposed model and perform modeling of bidirectional traffic with in IEEE 802.15.4 multi-hop networks for IoT applications.

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