MA30003/MA41003 - Linear Algebra

Problem Sheet 2 *

August 9, 2017

1 Span, Linear independence, Basis

Problem 1.1. *Show that if S_1 and S_2 are subsets of a vector space V such that $S_1 \subseteq S_2$, then $span(S_1) \subseteq span(S_2)$.*

Problem 1.2. *Show that if S_1 and S_2 are subsets of a vector space V, then $span(S_1 \cup S_2) = span(S_1) + span(S_2)$.*

Problem 1.3. *Show that the set $\{1, x, x^2, \dots, x^n, \dots\}$ is linearly independent in $\mathbb{P}((F))$.*

Problem 1.4. *Let $S = \{(1,1,0), (1,0,1), (0,1,1)\}$ be a subset of $\mathbb{F}^3$.*

1. *Prove that if $\mathbb{F} = \mathbb{R}$, then S is linearly independent.*
2. *Prove that if $\mathbb{F} = \mathbb{Z}_2$, then S is linearly dependent.*

Problem 1.5. *Prove that a set S of vectors is linearly independent if and only if each finite subset of S is linearly independent.*

Problem 1.6. *Let S be the set of nonzero polynomials in $\mathbb{P}(\mathbb{F})$ such that no two have the same degree. Prove that S is linearly independent.*

Problem 1.7. *Let V be a vector space. If the set $\{u_1, u_2\}$ is a basis, then the sets $\{u_1 + u_2, \alpha u_1\}$ and $\{\alpha u_1, \beta u_2\}$, where $\alpha, \beta \in \mathbb{F}$, is also a basis for V.*

Problem 1.8. *Consider the vector space $M_{n\times n}(\mathbb{R})$ over the field $\mathbb{R}$. Find a basis for the following subspaces of $M_{n\times n}(\mathbb{R})$ and hence the dimension.*

1. *the set of all symmetric matrices,*
2. *the set of all skew symmetric matrices,*
3. *the set of all upper triangular matrices,*
4. *the set of all matrices with trace zero.*

*Prepared by M. Rajesh Kannan. Please write to me at rajeshkannan1.m@gmail.com, rajeshkannan@maths.iitkgp.ernet.in, if you have any queries.

Problem 1.9. *Consider the vector space $\mathcal{F}(\mathbb{C}, \mathbb{C})$ over the field $\mathbb{C}$.*

1. *Prove that $\mathcal{F}(\mathbb{C}, \mathbb{C})$ is an infinite dimensional vector space.*
2. *Prove that the subspace of all even functions is an infinite dimensional subspace.*
3. *Prove that the subspace of all odd functions is an infinite dimensional subspace.*
4. *Prove that the subspace of all continuous functions is an infinite dimensional subspace.*

Problem 1.10. *Consider the vector space $\mathbb{P}(\mathbb{R})$ over the field $\mathbb{R}$. Find a basis for the subspaces of $\mathbb{P}(\mathbb{R})$.*

1. *the set of all odd polynomials,*
2. *the set of all even polynomials,*
3. $\{p(x) \in \mathbb{P}(\mathbb{R}) : p(0) = 0\}$,
4. $\{p(x) \in \mathbb{P}(\mathbb{R}) : p(1729) = p(1887) = 0\}$.