Some results on the smallest positive eigenvalue of trees

Sonu Rani*

June 8, 2021

Abstract

The smallest positive eigenvalue is an important parameter in mathematical chemistry. Let $\tau(T)$ denote the smallest positive eigenvalue of T. Let $\widehat{T}(v)$ be the tree obtained from tree T by attaching a pendant at vertex vof T. We show that if the smallest positive eigenvalue of $\widehat{T}(v)$ is lesser than that of T, then the row indexed by vertex v in the adjacency matrix of T is linearly dependent. We find the set U and W of vertices of a tree T such that $V(T) = U \cup W$ and $\tau(\widehat{T}(u)) \ge \tau(T)$ if $u \in U$; $\tau(\widehat{T}(u)) \le \tau(T)$ if $u \in W$. Also, we obtain a pair of noncospectral trees on same number of vertices having the same smallest positive eigenvalue.

Key words: Tree; adjacency matrix; smallest positive eigenvalue; matching AMS Subject Classification: 05C50; 05C05; 05C70

^{*}Rajiv Gandhi Govt College for Women, Bhiwani, Haryana, India 127021 (sr18@iitbbs.ac.in).