Resistance distance in directed cactus graphs and all minors matrix tree theorem

Shivani Goel

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Abstract

Let $G = (V, E)$ be a strongly connected and balanced digraph with vertex set $V = \{1, \ldots, n\}$. The classical distance d_{ij} between any two vertices i and j in G is the minimum length of all the directed paths joining i and j. The resistance distance (or, simply the resistance) between any two vertices i and j in V is defined by $r_{ij} := l^{\dagger}_{ii} + l^{\dagger}_{jj} - 2l^{\dagger}_{ij}$, where $l^{\dagger}_{pq}$ is the $(p, q)^{\mathrm{th}}$ entry of the Moore-Penrose inverse of L, which is the Laplacian matrix of G. In practice, the resistance r_{ij} is more significant than the classical distance. One reason for this is, numerical examples show that the resistance distance between i and j is always less than or equal to the classical distance, i.e. $r_{ij} \leq d_{ij}$. However, no proof for this inequality is known.

In a joint work with Dr. R. Balaji and Prof. R. B. Bapat, we show that this inequality holds for all directed cactus graphs. The proof uses a classical result from the literature known as "All minors matrix tree theorem". The main aim of this talk is to show the significance of all minors matrix tree theorem in proving the inequality.