A generalization of Fiedler's lemma and the spectra of H-join of graphs

Abstract

In this talk, we discuss the characteristic polynomial and spectra of H-join of graphs [2, 4]. In [5], the H-join operation of the graphs was initially introduced as generalized composition by Schwenk, denoted by $H[G_1, G_2, \ldots, G_k]$. Also, the same operation is studied in some other names as generalized lexicographic product and joined union. When all G_i's are equal to the same graph G, it is called the lexicographic product, denoted by $H[G]$.

Lemma: Let A be a symmetric $m \times m$ matrix with eigenvalues $\alpha_1, \alpha_2, \ldots, \alpha_m$ and B be a symmetric $n \times n$ matrix with eigenvalues $\beta_1, \beta_2, \ldots, \beta_n$. let u be an eigenvector of A corresponding to α_1 and v be an eigenvector of B corresponding to β_1 such that $\|u\| = \|v\| = 1$. Then for any constant ρ the matrix

$$C = \begin{bmatrix} A & \rho u v^t \\ \rho v u^t & B \end{bmatrix}$$

has eigenvalues $\alpha_2, \ldots, \alpha_m, \beta_2, \ldots, \beta_n, \gamma_1, \gamma_2$ where γ_1 and γ_2 are the eigenvalues of the matrix

$$\widehat{C} = \begin{bmatrix} \alpha_1 & \rho \\ \rho & \beta_1 \end{bmatrix}.$$

In [1, 2], the above lemma is called **Fiedler's lemma**. In [2, Theorem 3] Cardoso et al. obtained a generalization of Fiedler's lemma and hence obtained the spectra of H-join of regular graphs for any H. In [4], a new generalization of the Fiedler's lemma is obtained by introducing the concept of the main function of a matrix. As applications, the universal spectra of the H-join of any graphs (possibly non-regular) and the adjacency spectra of the H-generalized

1

join(introduced in [3]) constrained by (arbitrary) vertex subsets are obtained. The adjacency spectra of the generalized corona of graphs is deduced from the spectra of the H-join of graphs. Also, the construction of infinitely many pairs of non-regular universal cospectral graphs is provided. We discuss these results in the talk.

References

[1] D.M. Cardoso, I. Gutman, E.A. Martins, M. Robbiano *A generalization of Fiedler's lemma and some applications*, Linear Multilinear Algebra, 59(8) (2011) 929-942.

[2] D.M. Cardoso, M.A. de Freitas, E.A. Martins, M. Robbiano, *Spectra of graphs obtained by a generalization of the join graph operation*, Discrete Math., 313 (2013) 733-741.

[3] D.M. Cardoso, E.A. Martins, M. Robbiano, O. Rojo *Eigenvalues of a H-generalized join graph operation constrained by vertex subsets*, Lin. Alg. Appl., 438(8) (2013) 3278-3290.

[4] M. Saravanan, S. P. Murugan, G. Arunkumar, *A generalization of Fiedler's lemma and the spectra of H-join of graphs*, Lin. Alg. Appl. 625(8) 20-43

[5] A.J. Schwenk, *Computing the characteristic polynomial of a graph*, in: R. Bary, F. Harary (Eds.), Graphs Combinatorics, in: Lecture Notes in Mathematics, Springer-Verlag, Berlin, 406 (1974) 153-172.