Adjacency matrices of complex unit gain graphs

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Abstract

Let $G = (V, E)$ be a simple, undirected, finite graph with the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and the edge set $E(G) \subseteq V \times V$. If two vertices v_i and v_j are adjacent, we write $v_i \sim v_j$, and the edge between them is denoted by e_{ij}. The *adjacency matrix* of G is an $n \times n$ matrix, denoted by $A(G) = (a_{ij})$, whose rows and columns are indexed by the vertex set of the graph and the entries are defined by

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \sim v_j, \\ 0 & \text{otherwise.} \end{cases}$$

The adjacency matrix of a graph is one of the well studied matrix class in the field of spectral graph theory [1, 3, 4].

The notion of gain graph was studied in [10, 11]. For a given graph G and a group $\mathfrak{G}$, first orient the edges of the graph G. For each oriented edge e_{ij} assign a value (the *gain* of the edge e_{ij}) g from $\mathfrak{G}$ and assign g^{-1} to the orientated edge e_{ji}. If the group is taken to be the multiplicative group of unit complex numbers $\mathbb{T}$, the graph is called the *complex unit gain graph(or* $\mathbb{T}$*-gain graph).* In [9], the author defined the notion of adjacency matrices of $\mathbb{T}$-gain graphs, which is a canonical extension of classical adjacency matrices. Particular cases of the adjacency matrix of $\mathbb{T}$-gain graphs were considered with different gains in the literature [2, 5, 6, 7].

In this talk, we shall discuss some of the properties of adjacency matrices of graphs and adjacency matrices of $\mathbb{T}$-gain graphs[8].

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