Poincaré Series of Divisors on a Finite Graph

Madhusudan Manjunath

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Abstract

We study Poincaré series associated to a finite collection of divisors on a finite graph. Our main result is a proof of rationality of the Poincaré series. For a finite graph, our main technique involves studying a certain homomorphism from a free Abelian group of finite rank to the direct sum of the Jacobian of the graph and the integers. We express the Poincaré series as a finite integer combination of lattice point enumerating functions of rational polyhedra. Along the way, we review Riemann-Roch theory on a finite graph due to Baker and Norine.