Signed Distance in Signed Graphs

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Abstract

A signed graph is an ordered pair $\Sigma = (G, \sigma)$, where $G = (V, E)$ is the underlying graph of Σ with a signature function $\sigma : E \to \{1, -1\}$. Hence, signed graphs have their edges labeled either as positive or negative. Here we introduce two types of signed distance matrix for signed graph Σ. First we define auxiliary signs:

(S1) $\sigma_{\max}(u, v) = -1$ if all shortest uv-paths are negative, and $+1$ otherwise.

(S2) $\sigma_{\min}(u, v) = +1$ if all shortest uv-paths are positive, and -1 otherwise.

Then we define signed distances:

(d1) $d_{\max}(u, v) = \sigma_{\max}(u, v)d(u, v) = \max\{\sigma(P_{(u,v)}) : P_{(u,v)} \in \mathcal{P}_{(u,v)}\}d(u, v)$.

(d2) $d_{\min}(u, v) = \sigma_{\min}(u, v)d(u, v) = \min\{\sigma(P_{(u,v)}) : P_{(u,v)} \in \mathcal{P}_{(u,v)}\}d(u, v)$.

Finally, we define the signed distance matrices:

(D1) $D^{\max}(\Sigma) = (d_{\max}(u, v))_{n\times n}$.

(D2) $D^{\min}(\Sigma) = (d_{\min}(u, v))_{n\times n}$.

We characterize balance in signed graphs using these matrices and we obtain explicit formulae for the distance spectrum of some unbalanced signed graphs. We also introduce the notion of distance-compatible signed graphs and partially characterize it.

Key Words: Signed graph, Signed distance, Distance compatibility

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