Distance Spectra and Energy of Graphs

Anu Varghese
Bishop Chulaparambil Memorial College
Kottayam, Kerala.
(anukarintholil@gmail.com)

Abstract

The distance matrix of a connected graph G is an $n \times n$ matrix $D(G) = [d_{ij}]$, where d_{ij} is the distance between the vertices v_i and v_j. The distance spectrum of G is $\{\lambda_1, \ldots, \lambda_n\}$, where $\lambda_i's$ are the eigenvalues of $D(G)$. The distance energy of G is defined by $E_D(G) = \sum_{i=1}^{n} |\lambda_i|$. The distance matrix, distance eigenvalue and distance energy of a connected graph have been studied intensively in literature, see [1, 7, 8, 9, 12]. We discuss a new problem of how the distance energy changes when an edge is deleted. Similar problem for adjacency energy of a graph was studied by Day and So in [3, 4]. It turns out that the results for distance energy change and adjacency energy change are quite different. From an observation in [12], it follows that, for any connected graph with a unique positive eigenvalue, the deletion of any edge increases the distance energy provided that the resulting graph is still connected.

We prove that the distance energy of a complete bipartite graph is always increased when an edge is deleted even though it has two positive distance eigenvalues. Also, we give a set of examples of connected graph whose distance energy decreases when a specific edge is deleted.

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