



INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR  
MA20013 - Discrete Mathematics  
Problem Sheet 2 Spring 2019

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Problem 1. Prove that the number of different permutations of  $n$  objects, where there are  $n_1$  indistinguishable objects of type 1,  $n_2$  indistinguishable objects of type 2, ..., and  $n_k$  indistinguishable objects of type  $k$ , is  $\frac{n!}{n_1!n_2!\dots n_k!}$ .

Problem 2. How many solutions are there to the equation  $x_1 + x_2 + x_3 + x_4 = 17$ , where  $x_1, x_2, x_3$  and  $x_4$  are nonnegative integers.

Problem 3. How many solutions are there to the equation  $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ , where  $x_i$ ,  $i = 1, 2, 3, 4, 5$ , is a nonnegative integer so that

- (a)  $x_i \geq 1$ ?
- (b)  $x_i \geq 2$  for  $i = 1, 2, 3, 4, 5$ ?
- (c)  $0 \leq x_i \leq 10$ .
- (d)  $0 \leq x_1 \leq 3, 1 \leq x_2 \leq 4$ , and  $x_3 \geq 15$ ?

Problem 4. Show that the sequence  $\{a_n\}$  is a solution of the recurrence relation  $a_n = -3a_{n-1} + 4a_{n-2}$  if

- (a)  $a_n = 0$ .
- (b)  $a_n = 1$ .
- (c)  $a_n = (-4)^n$ .
- (d)  $a_n = 2(-4)^n + 3$ .

Problem 5. Find the solution of the recurrence relations with the initial conditions.

- (a)  $a_n = -a_{n-1}, a_0 = 5$ .
- (b)  $a_n = a_{n-1} + 3, a_0 = 1$ .
- (c)  $a_n = a_{n-1} - n, a_0 = 4$ .
- (d)  $a_n = 2na_{n-1}, a_0 = 3$ .

Problem 6. Solve these recurrence relations together with the initial conditions given.

- (a)  $a_n = a_{n-1} + 6a_{n-2}$  for  $n \geq 2, a_0 = 3, a_1 = 6$ .

(b)  $a_n = 7a_{n-1} - 10a_{n-2}$  for  $n \geq 2$ ,  $a_0 = 2$ ,  $a_1 = 1$ .

(c)  $a_n = -6a_{n-1} - 9a_{n-2}$  for  $n \geq 0$ ,  $a_0 = 2$ ,  $a_1 = 8$ .

Problem 7. Find the generating function for the sequence  $\{a_n\}$ , where

1.  $a_n = 5$  for all  $n = 0, 1, 2, \dots$ ,

2.  $a_n = 3^n$  for all  $n = 0, 1, 2, \dots$ ,

3.  $a_n = \binom{n+4}{n}$  for  $n = 0, 1, 2, \dots$ ,

4.  $a_n = \binom{10}{n+1}$ .

Problem 8. Use generating functions to determine the number of different ways 10 balloons can be given to four children if each child receives at least two balloons.

Problem 9. In how many ways can 25 identical pens be distributed to four students so that each student gets at least three but no more than seven pens?

Problem 10. Find the generating function for the sequence  $\{c_k\}$ , where  $c_k$  is the number of ways to make change for  $k$  rupees using 1 rupees, 2 rupees, 5 rupees, and 10 rupees coins?

Problem 11. What is the generating function for  $\{a_k\}$ , where  $a_k$  is the number of solutions of  $x_1 + x_2 + x_3 = k$ , where  $x_1, x_2$  and  $x_3$  are integer such that  $x_1 \geq 3$ ,  $0 \leq x_2 \leq 3$ , and  $2 \leq x_3 \leq 5$ ?

Problem 12. Find the exponential generating function for the following sequences:

1.  $a_n = \frac{1}{n+1}$ ,

2.  $a_n = n(n-1)$ ,

3.  $a_n = \frac{1}{(n+1)(n+2)}$ .