



INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR  
MA20013 - Discrete Mathematics  
Problem Sheet 1 Spring 2019

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Problem 1. If  $S$  is a nonempty set, then show that  $S$  has  $2^{|S|}$  distinct subsets.

Problem 2. Let  $A$  and  $B$  be two finite non-empty sets. How many functions are there between the sets  $A$  and  $B$ ?

Problem 3. A palindrome is a string whose reversal is identical to the string. How many bit strings of length  $n$  are palindromes?

Problem 4. Let  $d$  be a positive integer. Show that among any group of  $d+1$  (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by  $d$ .

Problem 5. If  $f$  is a function from  $S$  to  $T$ , where  $S$  and  $T$  are finite sets with  $|S| > |T|$ , then show that there exist two distinct elements  $x, y$  in  $S$  so that  $f(x) = f(y)$ .

Problem 6. Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equals to 9. Is the conclusion true if four integers selected rather than five integers.

Problem 7. Show that in a group of 5 people, there are not necessarily three mutual known people or mutually unknown people.

Problem 8. Let  $n$  and  $r$  be non-negative integers with  $r \leq n$ . Then, show that

$$\binom{n+1}{k+1} = \sum_{j=r}^n \binom{j}{r}.$$

Problem 9. Show that if  $p$  is a prime and  $k$  is an integer such that  $1 \leq k \leq (p-1)$ , then  $p$  divides  $\binom{p}{k}$ .

Problem 10. Prove that

$$\binom{n}{r} \binom{r}{k} = \binom{n}{k} \binom{n-k}{r-k},$$

whenever  $n$ ,  $r$ , and  $k$  are nonnegative integers with  $r \leq n$  and  $k \leq r$ .