INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

MA4050 - Combinatorics and Graph Theory

Problem Sheet 2 Autumn 2024

Problem 1. Find the trees that have the following Prüfer sequences:

(a) $(4, 3, 2, 3, 1)$.

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(a) $(1, 2, 1, 2, 1)$.

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Problem 2. Determine all the trees whose Prüfer sequences are constant.

Problem 3. Let $T_1, T_2, \ldots, T_k$ be subtrees of a tree T. Show that if $T_i \cap T_j \neq \emptyset$ for all i, j, then $\cap_{i=1}^{k} T_i \neq \emptyset$.

Problem 4. Let G be a graph with n vertices. Show that, if G contains no triangle then G has at most $\frac{n^2}{4}$ edges.

Problem 5. Let $G = A \cup B$ be a bipartite graph. If $|N(S)| \geq |S|$ for all $S \subseteq A$, then show that G has a matching containing all the vertices of A.

Problem 6. Show that every Eulerian bipartite graph have an even number of edges.

Problem 7. Prove that in any connected graph G, there is a walk that uses each edge exactly twice.

Problem 8. (a) Find the values of n such that K_n is Eulerian.

(b) Find the values of m and n such that $K_{m,n}$ is Eulerian.

Problem 9. (a) Find the values of n such that K_n is Hamiltonian.

(b) Find the values of m and n such that $K_{m,n}$ is Hamiltonian.

Problem 10. Show that the Petersen graph is not planer.

Problem 11. Consider the n-cube graph Q_n with the vertex set $\{0, 1\}^n$ defined as follows: Two vertices $(u_1, \ldots, u_n)$ and $(v_1, \ldots, v_n)$ are adjacent if and only if they differ exactly in one coordinate.

(a) Find the order, the size and the degree sequence of Q_n.

(b) Find all the values of n such that Q_n is Eulerian.

(c) Find all the values of n such that Q_n is Hamiltonian.

Problem 12. Let G be a graph that has exactly two connected components, both of them Hamiltonian graphs. Find the minimum number of edges that one needs to add to G to obtain a Hamiltonian graph.

Problem 13. Let G be a graph of odd order such that G and G^c are connected. Prove that G is Eulerian if and only if G^c is Eulerian.

Problem 14. Show that the graphs obtained from $K_{3,3}$ and K_5 by removing one edge are planer.

Problem 15. Determine all m and n so that $K_{m,n}$ is planar.