



INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD
MA5010/MA1240 - Combinatorics and Graph Theory/Combinatorics
Problem Sheet 5 Autumn 2023

Problem 1. Prove that the number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, ..., and n_k indistinguishable objects of type k , is $\frac{n!}{n_1!n_2!\dots n_k!}$.

Problem 2. Find the solution for the following recurrence relations:

1. $x_n = 6x_{n-1} - 9x_{n-2}$ with initial conditions $x_0 = 2, x_1 = 3$.

2. $x_n = 2x_{n-1} - 5x_{n-2}, n \geq 2$ with initial conditions $x_0 = 1, x_1 = 5$.

Problem 3. Find the solution for the following recurrence relation:

$$x_n = 15x_{n-2} - 10x_{n-3} - 60x_{n-4} + 72x_{n-5}$$

with the initial conditions

$$x_0 = 1, x_1 = 6, x_2 = 9, x_3 = -110, x_4 = -45.$$

Problem 4. Solve the following non-homogenous recurrence relation:

$$x_n = 3x_{n-1} + 10x_{n-2} + 7 \times 5^n$$

with the initial conditions:

$$x_0 = 4, x_1 = 3.$$

Problem 5. Find a generating function for the sequence 1, 3, 5, 7, 9, ...

Problem 6. Find a generating function for the sequences:

1. 1, 4, 9, 16, ...

2. 5, 5, 5, 5, ...

3. 0, 0, 0, 2, 4, 6, 8, ...

4. 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0, 5, ...

Problem 7. Find the sequence generated by the following generating functions:

1. $a_n = 3a_{n-1} + z_{n-2}$ with the initial conditions $a_0 = 1$ and $a_1 = 5$.

2. Find a generating function for the Fibonacci numbers.

3. $a_n = 2 \times 5^n + 7 \times (-3)^n$

Problem 8. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$, where x_1, x_2, x_3 and x_4 are nonnegative integers.

Problem 9. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$, where $x_i, i = 1, 2, 3, 4, 5$, is a nonnegative integer so that

(a) $x_i \geq 1$?

(b) $x_i \geq 2$ for $i = 1, 2, 3, 4, 5$?

(c) $0 \leq x_i \leq 10$.

(d) $0 \leq x_1 \leq 3, 1 \leq x_2 \leq 4$, and $x_3 \geq 15$?

Problem 10. Determine the number of integer solutions for the equation

$$x_1 + x_2 + x_3 + x_4 \leq 38,$$

where

1. $x_i \geq 0$ for $1 \leq i \leq 4$.

2. $x_1 \geq 0, x_2 \geq 2, x_3 \geq -2, 3 \leq x_4 \leq 8$

Problem 11. Use generating functions to determine the number of different ways 10 balloons can be given to four children if each child receives at least two balloons.

Problem 12. In how many ways can 25 identical pens be distributed to four students so that each student gets at least three but no more than seven pens?

Problem 13. Find the generating function for the sequence $\{c_k\}$, where c_k is the number of ways to make change for k rupees using 1 rupees, 2 rupees, 5 rupees, and 10 rupees coins?

Problem 14. What is the generating function for $\{a_k\}$, where a_k is the number of solutions of $x_1 + x_2 + x_3 = k$, where x_1, x_2 and x_3 are integer such that $x_1 \geq 3, 0 \leq x_2 \leq 3$, and $2 \leq x_3 \leq 5$?

Problem 15. Find the exponential generating function for the following sequences:

1. $a_n = \frac{1}{n+1}$,

2. $a_n = n(n-1)$,

3. $a_n = \frac{1}{(n+1)(n+2)}$.

Problem 16. How many ways are there to triangulate a regular convex n -gon, if two triangulations are regarded as being the same if they can be made to coincide by a rotation of the polygon?