INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD
MA5010 - Combinatorics and Graph Theory
Problem Sheet 4 Autumn 2023

Problem 1. What is the number of all compositions of n in which the first part is not 2?

Problem 2. What is the number of all weak compositions of 10 into five parts so that exactly two parts are 0?

Problem 3. Find the number of compositions of n into an even number of parts.

Problem 4. Find the number of weak compositions of 25 into five odd parts.

Problem 5. Prove that $S(n,2) < 2^{n-1} - 1$.

Problem 6. Prove that $S(n, n-2) < C(n,3) + 3C(n,4)$.

Problem 7. Prove that $n! < S(2n, n) < (2n)!$

Problem 8. For all $m \geq 1, n \geq 0, m \geq n$, prove that $S(m,n) = \frac{1}{n!}\sum_{k=0}^{n}(-1)^{n-k}C(n,k)k^m$.

Problem 9. Prove that if $n \geq 3$, then $B(n) < n!$

Problem 10. Prove that if $n \geq 1$, then $B(n) = \sum_{k=0}^{n-1} C(n-1,k)B(k)$.

Problem 11. Prove that if $n \geq 1$, then $B(n) = \frac{1}{e}\sum_{j=0}^{\infty}\frac{j^j}{j!}$.

Problem 12. Prove that the number of partitions of n into exactly k parts is equal to the number of partitions of n in which the larger part is exactly k.

Problem 13. Prove that the number of partitions of n into at most k parts is equal to that of the partitions of $(n+k)$ into exactly k parts.

Problem 14. Prove that for all integers $n \geq 2$, the number $p(n) - p(n-1)$ is equal to the number of partitions of n in which the two larger parts are equal.

Problem 15. Find the number of compositions of n in which the i^{th} part is equal to k.