INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

MA5010/MA1240 - Combinatorics and Graph Theory/Combinatorics

Problem Sheet 2 Autumn 2023

Problem 1. (a) Find the values of n such that K_n is Eulerian.

(b) Find the values of m and n such that $K_{m,n}$ is Eulerian.

Problem 2. (a) Find the values of n such that K_n is Hamiltonian.

(b) Find the values of m and n such that $K_{m,n}$ is Hamiltonian.

Problem 3. Show that the Petersen graph is not planer.

Problem 4. Consider the n-cube graph Q_n with the vertex set $\{0,1\}^n$ defined as follows: Two vertices $(u_1, \dots, u_n)$ and $(v_1, \dots, v_n)$ are adjacent if and only if they differ exactly in one coordinate.

(a) Find the order, the size and the degree sequence of Q_n.

(b) Find all the values of n such that Q_n is Eulerian.

(c) Find all the values of n such that Q_n is Hamiltonian.

Problem 5. Let G be a graph that has exactly two connected components, both of them Hamiltonian graphs. Find the minimum number of edges that one needs to add to G to obtain a Hamiltonian graph.

Problem 6. Find the trees that have the following Prüfer sequences:

(a) $(4, 3, 2, 3, 1)$.

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(a) $(1, 2, 1, 2, 1)$.

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Problem 7. Determine all the trees whose Prüfer sequences are constant.

Problem 8. Let G be a graph of odd order such that G and G^c are connected. Prove that G is Eulerian if and only if G^c is Eulerian.

Problem 9. Show that the graphs obtained from $K_{3,3}$ and K_5 by removing one edge are planer.

Problem 10. Determine all m and n so that $K_{m,n}$ is planar.