

Problem Sheet – 2

Improper Integrals

- No need to submit the answers to the instructors.

1. Evaluate the following improper integrals.

(a) $\int_{-1}^1 \frac{dx}{x^{2/3}}$

(b) $\int_{-8}^1 \frac{dx}{x^{1/3}}$

(c) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(d) $\int_0^1 \frac{dr}{r^{0.999}}$

(e) $\int_{-\infty}^{-2} \frac{2 dx}{x^2 - 1}$

(f) $\int_{-\infty}^2 \frac{2 dx}{x^2 + 4}$

(g) $\int_0^{\infty} \frac{dx}{(1+x)\sqrt{x}}$

(h) $\int_1^{\infty} \frac{1}{x\sqrt{x^2-1}} dx$

(i) $\int_0^{\infty} \frac{dv}{(1+v^2)(1+\tan^{-1} v)}$

(j) $\int_0^{\infty} \frac{16 \tan^{-1} x}{1+x^2} dx$

(k) $\int_{-\infty}^0 \theta e^{\theta} d\theta$

(l) $\int_0^{\infty} 2e^{-\theta} \sin \theta d\theta$

(m) $\int_{-\infty}^0 e^{-|x|} dx$

(n) $\int_{-\infty}^{\infty} 2xe^{-x^2} dx$

2. Discuss the convergence of the following improper integrals:

(a) Evaluate the following improper integrals.

i. $\int_0^1 \frac{\ln x}{x^2} dx$

ii. $\int_1^2 \frac{dx}{x \ln x}$

iii. $\int_0^{\ln 2} x^{-2} e^{-1/x} dx$

iv. $\int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$

v. $\int_1^{\infty} \frac{dx}{x^3 + 1}$

vi. $\int_4^{\infty} \frac{dx}{\sqrt{x}-1}$

vii. $\int_2^{\infty} \frac{dv}{\sqrt{v}-1}$

viii. $\int_0^{\infty} \frac{d\theta}{1+e^{\theta}}$

ix. $\int_{\pi}^{\infty} \frac{2 + \cos x}{x} dx$

x. $\int_{\pi}^{\infty} \frac{1 + \sin x}{x^2} dx$

3. Show that

$$\int_0^{\infty} \frac{2x}{x^2 + 1} dx$$

diverges and hence that

$$\int_{-\infty}^{\infty} \frac{2x}{x^2 + 1} dx$$

diverges. Then show that

$$\lim_{b \rightarrow \infty} \int_{-b}^b \frac{2x}{x^2 + 1} dx = 0.$$

This shows that

$$\int_{-\infty}^{\infty} f(x) dx \quad \text{may not equal} \quad \lim_{a \rightarrow \infty} \int_{-a}^a f(x) dx.$$