Problem Set - 8

Spring 2018

MATHEMATICS-II (MA10002)

1. (a) Find the jacobian of the following transformations T.

   (i) $T : x + y = u,\ y = uv$. Find $J = \dfrac{\partial(x, y)}{\partial(u, v)}$.

   (ii) $T : x = 2u + 3v,\ y = 2u - 3v$. Find $J = \dfrac{\partial(x, y)}{\partial(u, v)}$.

   (iii) $T : x + y + z = u,\ x + y = uv,\ x = uvw$. Find $J = \dfrac{\partial(x, y, z)}{\partial(u, v, w)}$.

   (iv) $T : x = r \cos\phi \sin\theta,\ y = r \sin\phi \sin\theta,\ z = r \cos\theta$. Find $J = \dfrac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$.

   (b) Evaluate the double integrations using change of variable.

   (i) Evaluate $\displaystyle\iint_R \sqrt{x^2 + y^2}\, dxdy$, the field of integration being R, the region in xy plane bounded by the circle $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. [Hint: $x = r\cos\theta$, $y = r\sin\theta$.]

   (ii) Using the transformation $x+y = u$, $y = uv$, show that $\displaystyle\iint_E e^{\frac{y}{x+y}}\, dxdy = \frac{1}{2}(e-1)$ where E is the triangle bounded by $x = 0$, $y = 0$, $x + y = 1$.

   (iii) Evaluate $\displaystyle\iint_R (x + y)\, dA$, where R is the trapezoidal region with vertices given by $(0, 0)$, $(5, 0)$, $(\frac{5}{2}, \frac{5}{2})$ and $(\frac{5}{2}, -\frac{5}{2})$ using the transformation $x = 2u + 3v$ and $y = 2u - 3v$.

   (c) Evaluate

   $$\iint_R \frac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}}\, dxdy,$$

   the field of integration being R, the positive quadrant of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. [Hint: change ellipse to a circle using $x = au, y = bv$.]

2. Show that $\displaystyle\iint_E y\, dxdy = \frac{1}{3}a^3 - \frac{a^2}{2}k + \frac{b}{4}k^2 + \frac{1}{6}k^3$ where $k = -\frac{b}{2} + \sqrt{a^2 + \frac{b^2}{4}}$ and E is the region in the first quadrant bounded by x-axis, the curves $x^2 + y^2 = a^2$, $y^2 = bx$.

3. Find the value of the following triple integrals.

   a) $\displaystyle\iiint_R (x + y + z)\, dxdydz$ where $R : 0 \le x \le 1,\ 1 \le y \le 2,\ 2 \le z \le 3$.

   b) $\displaystyle\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\, dzdydx$.

4. Compute $\iiint \frac{dxdydz}{(1+x+y+z)^3}$ if the region of integration is bounded by the co-ordinate planes and the plane $x+y+z=1$.

5. Evaluate $\iiint x^2yz\,dxdydz$ throughout the volume bounded by the planes $x=0$, $y=0$, $z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, by using $x=av$, $y=bv$, $z=cw$.

6. Using spherical co-ordinate evaluate $\iiint (x^2+y^2+z^2)\,dxdydz$ enclosed by the sphere $x^2+y^2+z^2=1$.

7. Evaluate $\iiint_R y\,dV$ where R is the region lies below the plane $z=x+1$ above the xy-plane and between the cylinders $x^2+y^2=1$ and $x^2+y^2=4$.

8. Find the surface area of the cylinder $x^2+z^2=4$ inside the cylinder $x^2+y^2=4$.

9. Find the surface area of the sphere $x^2+y^2+z^2=9$ lying inside the cylinder $x^2+y^2=3y$.

10. Find the surface area of the section of the cylinder $x^2+y^2=a^2$ made by the plane $x+y+z=a$.

11. Find the volume of the solid bounded by the parabolic $y^2+z^2=4x$ and the plane $x=5$.

12. Calculate the volume of the solid bounded by the following surfaces

$$z=0,\ x^2+y^2=1,\ x+y+z=3.$$

13. Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$.

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