Tutorial Sheet - 8

SPRING 2017

MATHEMATICS-II (MA10002)

January 2, 2017

1. Discuss the convergence of improper integrals using definition:

   (i) $\displaystyle\int_1^\infty \frac{1}{x^4}dx$ (ii) $\displaystyle\int_{10}^\infty \frac{1}{x\ln x}dx$ (iii) $\displaystyle\int_0^1 \frac{\ln x}{x^2}dx$

   (iv) $\displaystyle\int_{-\infty}^\infty \frac{x}{(x^2+1)}dx$ (v) $\displaystyle\int_1^2 \frac{4x}{(x^2-4)^{\frac{1}{3}}}dx$ (vi) $\displaystyle\int_1^\infty \frac{x+1}{x^{\frac{3}{2}}}dx$

   (vii) $\displaystyle\int_{-\infty}^0 \frac{1}{(3-x)^{\frac{1}{2}}}dx$ (viii) $\displaystyle\int_{-2}^\infty \sin(x)dx$ (ix) $\displaystyle\int_1^\infty \frac{1}{(1+x)\sqrt{x}}dx$

   (x) $\displaystyle\int_1^2 \frac{1}{x\ln^2 x}dx$

2. Discuss the convergence of the following integrals :

   (i) $\displaystyle\int_0^1 \frac{1}{x^2+\sqrt{x}}dx$ (ii) $\displaystyle\int_0^\infty \frac{1}{x+e^x}dx$ (iii) $\displaystyle\int_0^\infty \frac{1}{x^2+xe^x}dx$

   (iv) $\displaystyle\int_0^\infty \frac{1-\cos(x)}{x^2}dx$ (v) $\displaystyle\int_1^\infty \frac{x}{(1+x)^3}dx$ (vi) $\displaystyle\int_1^\infty \frac{x}{3x^4+5x^2+1}dx$

   (vii) $\displaystyle\int_{-\infty}^\infty e^{-|x|}dx$ (viii) $\displaystyle\int_0^\infty \frac{\cos x}{e^x}dx$ (ix) $\displaystyle\int_1^\infty e^{x+x^{-1}}dx$

   (x) $\displaystyle\int_0^1 \frac{e^x}{x^2}dx$

3. Examine the convergence of the following integrals :

   (i) $\displaystyle\int_0^1 \frac{1}{(x+2)\sqrt{x(1-x)}}dx$ (ii) $\displaystyle\int_0^\infty x^{-\frac{1}{2}}e^{-x}dx$ (iii) $\displaystyle\int_1^\infty \frac{1}{x^{\frac{1}{2}}(1+x)^{\frac{1}{4}}}dx$

   (iv) $\displaystyle\int_0^\infty \frac{\cos(x)}{\sqrt{x^3+x}}dx$ (v) $\displaystyle\int_0^1 \frac{x^{p-1}}{1-x}dx$

4. Prove that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x^m}{\sin(x)^n}dx$ is convergent iff $n < m+1$

5. Show that the improper integral $\int\limits_0^1 \frac{\sin(\frac{1}{x})}{\sqrt{x}}dx$ is convergent.

6. Prove that the integral $\int\limits_0^\infty \left(\frac{1}{x+1} - \frac{1}{e^x}\right)\frac{1}{x}dx$ is convergent

7. Test the convergent of $\int\limits_0^\infty e^{-x^2}dx$

8. Explain the convergence of $\int\limits_0^1 \frac{\ln x}{\sqrt{x}}dx$

9. Show that $\int\limits_0^1 x^{m-1}(1-x)^{n-1}dx$ is convergent iff m, n are both positive.

10. Show that $\int\limits_0^\infty \frac{\tan^{-1}(ax) - \tan^{-1}(bx)}{x}dx = \frac{\pi}{2}\log(\frac{a}{b}) \quad 0 < b < a$

11. Prove that $\int\limits_0^\infty \frac{\sin(x)(1-\cos(x))}{x^2}dx = \log 2$