Tutorial Sheet - 6

SPRING 2017

MATHEMATICS-II (MA10002)(Numerical Analysis) *January 5, 2017*

1. For the following data, find a polynomial $f(x)$ and hence find the value of $f(1.5)$

x	1	2	3	4	5
$f(x)$	4	13	34	73	136

2. Find the missing terms in the following table

x	0	1	2	3	4	5
$f(x)$	0	-	8	15	-	35

3. In an examination the number of candidates who secured marks between certain limit were as follows:

x	0-19	20-39	40-59	60-79	80-99
$f(x)$	41	62	65	50	17

   Estimate the number of candidate getting marks less than 70.

4. Compute $f(21)$ using Newton's backward difference formula from the following data

x	0	5	10	15	20
$f(x)$	1.0	1.6	3.8	8.2	15.4

5. Find $f(1.02)$ using Newton forward difference formula from the following table

x	1.00	1.10	1.20	1.30
$f(x)$	0.8415	0.8912	0.9320	0.9636

6. Find the Lagrange's interpolating polynomial satisfying the following data

x	-1	0	2	5
$f(x)$	9	5	3	15

7. Prove the following properties:
   (a) $\Delta \cdot \nabla = \Delta - \nabla$
   (b) $E \cdot \Delta = \Delta \cdot E$
   (c) $E = I + \Delta$
   where Δ, ∇, E and I are forward difference, backward difference, shift and identity operator, respectively.

8. Evaluate the integral $\int_{0.1}^{0.7}(e^x + 2x)$ by taking $h = 0.5$, correct up to 5-decimal places by
(a) Trapezoidal rule
(b) Simpson's $\frac{1}{3}$ rule.

9. Write down the linear function which takes the same values as $f(x)$ at $x = x_0, x_1$ and integrate it to obtain the Trapezoidal rule for approximation of $f(x)$ over $(0, 1)$. Prove that the error is $-\frac{h^3}{12}f''(\xi)$, where $h = x_1 - x_0$ and $x_0 < \xi < x_1$.

10. Evaluate $\int_1^2 \frac{dx}{x}$, taking 4-sub intervals, correct up to five decimal places by
(a) Trapizoidal rule
(b) Simson's $\frac{1}{3}$ rule
Also find the absolute error.

11. Let $f(x) = \ln(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use linear Lagrange interpolation to calculate an approximate value $f(1.04)$ and obtain an bound on the truncation error.

12. Determine the appropriate step size to use, in the construction of a table of $f(x) = (1 + x)^6$ on [0,1]. The truncation error for linear interpolation is to be bounded by 5×10^{-5}

13. (a) Show that the truncation error of quadratic interpolation in an equidistant table is bounded by

$$\left(\frac{h^3}{9\sqrt{3}}\right) max|f'''(\xi)|$$

(b) We want to set up an equidistant table of $f(x) = x^2 \ln(x)$ in the interval $5 \leq x \leq 10$. The function values are rounded to 5 decimals. Give the step size h which is to be used to yield a total error less than 10^{-5} on quadratic Lagrange interpolation in this table.