Tutorial Sheet - 3

SPRING 2017

MATHEMATICS-II (MA10002) *January 10, 2017*

1. Find the rank of the matrix A using definition where A is given in the following.
$(a)\ \begin{pmatrix} 1 & 0 & 3 \\ 4 & -1 & 5 \\ 2 & 0 & 6 \end{pmatrix}$ $(b)\ \begin{pmatrix} 2 & 3 & -1 & 1 \\ 3 & 0 & 4 & 2 \\ 6 & 9 & -3 & 3 \end{pmatrix}$

2. Determine the rank of the following matrices by reducing to row echelon form
$(a)\ \begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$ $(b)\ \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 3 & 6 & 6 & 3 \end{pmatrix}$ $(c)\ \begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$

3. Find all x such that the rank of the matrix $\begin{pmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{pmatrix}$ is less than 3.

4. If the distinct roots α, β, γ of the equation $x^3 + qx + r = 0$ are in Arithmetic Progression, then show that the rank of the matrix $\begin{pmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{pmatrix}$ is 2.

5. Using Gauss elimination method find all possible solutions of the following system of linear equations.

(a)
$$\begin{aligned} x + y - z &= 0 \\ 2x - 3y + z &= 0 \\ x - 4y + 2z &= 0 \end{aligned}$$

(b)
$$\begin{aligned} x + y - z &= 0 \\ 2x + 4y - z &= 0 \\ 3x + 2y + 2z &= 0 \end{aligned}$$

(c)
$$\begin{aligned} x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 &= 2 \\ 3x_1 - 9x_2 + 7x_3 - x_4 + 3x_5 &= 7 \\ 2x_1 - 6x_2 + 7x_3 + 4x_4 - 5x_5 &= 7 \end{aligned}$$

6. Discuss the consistency of the system of equations and solve if possible.

(a)
$$\begin{aligned} x_1 + x_2 &= 4 \\ x_2 - x_3 &= 1 \\ 2x_1 + x_2 + 4x_3 &= 7 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 2x_2 - x_3 &= 10 \\ -x_1 + x_2 + 2x_3 &= 2 \\ 2x_1 + x_2 - 3x_3 &= 2 \end{aligned}$$

(c)
$$\begin{aligned} x_1 + 3x_2 + x_3 &= 0 \\ 2x_1 - x_2 + x_3 &= 0 \end{aligned}$$

7. Find the value of k for which the system of equations has non-trivial solution

(i)
$$\begin{aligned} x + y + z &= 0 \\ y + z &= 0 \\ ky + z &= 0 \end{aligned}$$

(ii)
$$\begin{aligned} (3k-8)x + 3y + 3z &= 0 \\ 3x + (3k-8)y + 3z &= 0 \\ 3x + 3y + (3k-8)z &= 0 \end{aligned}$$

8. Determine the conditions on a and b for which the following system of equations admit (i) unique solution (ii) no solution (iii) infinitely many solutions.

(a)
$$\begin{aligned} x+2y-z-t&=0\\ 2x+5y+z+t&=8\\ 3x+7y+2z+2t&=b\\ -x+z+at&=16 \end{aligned}$$

(b)
$$\begin{aligned} x+y+z&=1\\ x+2y-z&=b\\ 5x+7y+az&=b^2 \end{aligned}$$

(c)
$$\begin{aligned} x-y+z&=1\\ x+2y+4z&=a\\ x+4y+6z&=a^2 \end{aligned}$$

9. If the following system

$$\begin{aligned} ax+by+cz&=0\\ bx+ay+az&=0\\ cx+ay+bz&=0 \end{aligned}$$

has non trivial solution, then prove that $a+b+c=0$ or $a=b=c$.

10. Express the matrix $A=\begin{pmatrix} 1 & 2+i & 1-i \\ 2-i & 1+2i & 3 \\ 2+i & 2 & 1+i \end{pmatrix}$ as the sum of a Hermitian and a Skew-Hermitian matrix.

11. If $A=\begin{pmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{pmatrix}$, then show that AA^* is a Hermitian matrix, where A^* is conjugate transpose of A.

12. If A is real and non symmetric matrix of order 3, then prove that the rank of the matrix $A-A^T$ is 2.

13. Show that the matrix $\begin{pmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{pmatrix}$ is unitary matrix if $\alpha^2+\beta^2+\gamma^2+\delta^2=1$.

14. If $M=\begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$, where $a=e^{\frac{2i\pi}{3}}$, then prove that $M^{-1}=\frac{1}{3}\overline{M}$.