Tutorial Sheet - 10

SPRING 2017

MATHEMATICS-II (MA10002)

1. Evaluate $\int\int\int_D 2x dV$, where E is the region under the plane $2x + 3y + z = 6$ that lies in the first octant.

2. Evaluate $\int\int\int \frac{dxdydz}{(x+y+z+1)^3}$, over a tetrahedron bounded by coordinate planes and the plane $x + y + z = 1$.

3. Using spherical coordinates evaluate
(i) $\int\int\int_D (x^2 + y^2 + z^2)^m dxdydz$, $m > 0$, over the region $D = \{(x, y, z); x^2 + y^2 + z^2 \leq 1\}$.
(ii) $\int\int\int_D \sqrt{x^2 + y^2 + z^2} dxdydz$, D is the region bounded by the plane $z = 3$ and cone $z = \sqrt{x^2 + y^2}$.

4. Using cylindrical coordinates evaluate
(i) $\int\int\int_D \sqrt{x^2 + y^2} dxdydz$, where D is region lying above xy-plane and below cone $z = 4 - \sqrt{x^2 + y^2}$.
(ii) $\int\int\int_D y$, where D is the region that lies below the plane $z = x + 2$ above the xy-plane and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

5. Find the surface area of the cylinder $x^2 + z^2 = 4$ inside the cylinder $x^2 + y^2 = 4$.

6. Find the surface area of the section of the cylinder $x^2+y^2 = a^2$ made by the plane $x+y+z = a$.

7. Find the area of the part of the surface of the paraboloid $y^2 + z^2 = 2ax$ which lies between the cylinder $y^2 = ax$ and the plane $x = a$.

8. Find the volume bounded by the surfaces $z = 4 - x^2 - \frac{y^2}{4}$ and $z = 3x^2 + \frac{y^2}{4}$.

9. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

10. Find the volume cut off from the paraboloid $x^2 + \frac{y^2}{4} + z = 1$ by the plane $z = 0$.

11. Find the volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the paraboloid $x^2 + y^2 = 3z$

12. Evaluate
$$\int_0^{\frac{\pi}{2}} \log(\alpha \cos^2 \theta + \beta \sin^2 \theta) d\theta, \qquad (\alpha > 0, \beta > 0).$$

13. Evaluate

$$\int_0^\infty e^{-\alpha x}\frac{\sin\beta x}{x}dx, \qquad where \;\; \alpha \geq 0,$$

hence deduce that

$$\int_0^\infty \frac{sin\beta x}{x}dx = \begin{cases} \frac{\pi}{2}, & ift \;\; \beta > 0, \\ 0, & ift \;\; \beta = 0, \\ -\frac{\pi}{2}, & ift \;\; \beta < 0. \end{cases}$$

14. Show that

$$\int_0^\infty \frac{\tan^{-1}\alpha x \;\; \tan^{-1}\beta x}{x^2}dx = \frac{\pi}{2}log\Big[\frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha\beta^\beta}\Big]$$