MA20103 Partial Differential Equations
Assignment 6

1. (a) Given that the displacement, $u(x,t)$, of a flexible string is governed by the equation: $c^2u_{xx} = u_{tt}$, where c is a constant. The string which is initially at rest has one end fixed at $x = 0$ and the other end at $x = 1$. The string is set to motion by an initial shape of $x(1-x)$, however with zero velocity. Formulate the corresponding initial boundary value problem and obtain the solution using separation of variables subject to the boundary conditions $u(0,t) = u(1,t) = 0,\ t \geq 0$.

   Solve the following initial boundary value problems

   (b)
$$u_{tt} = c^2u_{xx}, \quad 0 < x < \pi, \quad t > 0,$$
$$u(x,0) = 0, \quad u_t(x,0) = 8\sin^2 x, \quad 0 \leq x \leq \pi,$$
$$u(0,t) = u(\pi,t) = 0, \quad t > 0.$$

   (c)
$$u_{tt} = c^2u_{xx}, \quad 0 < x < 1, \quad t > 0,$$
$$u(x,0) = 0, \quad u_t(x,0) = x\sin\pi x, \quad 0 \leq x \leq 1,$$
$$u(0,t) = u(1,t) = 0, \quad t > 0.$$

   (d)
$$u_{tt} = c^2u_{xx}, \quad 0 < x < 1, \quad t > 0,$$
$$u(x,0) = x(1-x), \quad u_t(x,0) = x - \tan\frac{\pi x}{4}, \quad 0 \leq x \leq 1,$$
$$u(0,t) = u(\pi,t) = 0, \quad t > 0.$$

   (e)
$$u_{tt} = c^2u_{xx}, \quad 0 < x < \pi, \quad t > 0,$$
$$u(x,0) = x + \sin x, \quad u_t(x,0) = 0, \quad 0 \leq x \leq \pi,$$
$$u(0,t) = u_x(\pi,t) = 0, \quad t > 0.$$

   (f)
$$u_{tt} = c^2u_{xx}, \quad 0 < x < \pi, \quad t > 0,$$
$$u(x,0) = \cos x, \ u_t(x,0) = 0, \quad 0 \leq x \leq \pi,$$
$$u_x(0,t) = 0, \quad u_x(\pi,t) = 0, \quad t > 0.$$

   (g) Consider the process of unsteady heat conduction in a rod of finite length $0 < x < 1$, which is governed by $u_t = ku_{xx}$, where $u(x,t)$ denote the temperature at a position x at time t. If the initial temperature is give by $x(1-x)$, $\ 0 \leq x \leq 1$, and the end $x = 0$ is kept at a temperature t while the other end $x = 1$ is kept at $\sin t$ for $t \geq 0$, formulate the initial boundary value problem and then obtain solution using separation of variables.

   (h) Solve
$$u_t = c^2u_{xx}, \qquad 0 \leq x \leq 1,$$
   subject to
$$u(0,t) = 2, \quad u(1,t) = 3, \quad u(x,0) = x(1-x).$$

2. (a) Solve the boundary value problem in an annular region:

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0, \quad a < r < b, \quad 0 < \theta < \alpha,$$

subject to

$$u(r,0) = 0, \quad u(r,\alpha) = 0, \quad u(a,\theta) = 0, \quad u(b,\theta) = 100.$$

(b) Solve the boundary value problem

$$u_{xx} + u_{yy} = 0, \quad 0 \le x \le a, \quad 0 \le y \le b,$$

subject to

$$(i) \;\; u(0,y) = 0, \quad u(a,y) = 0, \quad u_y(x,b) = 0, \quad u(x,0) = x^2.$$

$$(ii) \;\; u(0,y) = 0, \quad u(a,y) = 0, u(x,b) = b - x, \quad u(x,0) = 0.$$

****Good Luck****