

MA20103 - Partial differential equations Assignment

August 10, 2016

Submit the assignment on or before Wednesday 17 August 2016 at 8.00 am, late submissions will carry no marks. You can drop the assignment in my office N340.

1. (Problem 10, Page 179, Kreysig's book) Define generating function of a sequence. Show that

$$G(u, x) = \frac{1}{\sqrt{1 - 2xu + u^2}} = \sum_{n=0}^{\infty} P_n(x)u^n$$

is a generating function of the sequence of Legendre polynomials $P_0(x), P_1(x), \dots, P_n(x), \dots$

2. (Problem 14, Page 180, Kreysig's book)(Bonnet's formula) Prove that

$$(n + 1)P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x),$$

where $n = 1, 2, \dots$.(Using the generating function)

3. (Bonnet's formula) Prove that

$$(n + 1)P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x),$$

where $n = 1, 2, \dots$.(Using the Rodrigues's formula)

4. (Problem 13, Page 195, Kreysig's book)Using Rolle's theorem, show that between any two consecutive zeros of $J_n(x)$ there is precisely one zero of $J_{n+1}(x)$, where $J_n(x)$ denotes the Bessel function of first kind of order n .