

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 2

MA 4090 : Combinatorics

Max Marks: 65

1. Show that $S(8n, 8n - 2)$ and $s(8n, 8n - 2)$ are even numbers, for all n . [5]

2. Show that [5]

$$S(n, n - 2) = \binom{n}{3} + 3\binom{n}{4}.$$

3. Find a recurrence relation involving the terms $s(n, k)$, $s(n - 1, k - 1)$, and $s(n - 1, k)$. [5]

4. Calculate the numbers $S(5, k)$ for all k . [5]

5. Solve the recurrence relations:

(a) $a_n = 3a_{n-1} + 3a_{n-2} - a_{n-3}$ subjected to the conditions $a_0 = a_1 = 1$ and $a_2 = 2$ [2]

(b) $a_n = a_{n-1} + 9a_{n-2} - 9a_{n-3}$ subjected to the conditions $a_0 = 0, a_1 = 1$ and $a_2 = 2$ [3]

6. If the recurrence relation $a_n = ca_{n-1} + da_{n-2}$ has a general solution of the form $a_n = A3^n + B6^n$, then find the values of c and d . [5]

7. Solve the recurrence relation [5]

$$a_n = 5a_{n-1} - 6a_{n-2} - 4a_{n-3} + 8a_{n-4}$$

for $n = 4, 5, 6, \dots$ subjected to the initial conditions $a_0 = 0, a_1 = 1, a_2 = 1$ and $a_3 = 2$.

8. Let $f(x)$ be the generating function for the sequence $\{a_n\}_{n=0}^{\infty}$. Show that $f(x)$ has an inverse iff $a_0 \neq 0$. [5]

9. Let $f(x)$ be the generating function for the sequence $\{a_n\}_{n=0}^{\infty}$. What is the generating function for the following sequences?

(a) $(0, 0, 0, 0, a_0, a_1, a_2, \dots)$ [1]

(b) $(a_1, a_0, a_2, a_3, \dots)$ [2]

(c) $(a_0, -a_1, a_2, -a_3, \dots)$ [2]

10. The recurrence relation for $\{a_n\}_{n=1}^{\infty}$ is given by [5]

$$a_{n+1} = \begin{cases} 2a_n, & \text{if } n \text{ is odd,} \\ 2a_n + 1, & \text{if } n \text{ is even,} \end{cases}$$

with the initial condition $a_1 = 1$. Show that the recurrence relation can be written in the form $a_{n+2} = a_{n+1} + 2a_n + 1$ and then solve it.

11. Show that the generating function of the sequence $a_n = n^2$ is given by $\frac{x^2+x}{(1-x)^4}$. [5]

12. Let $p_k(n)$ denote the number of partitions of n into k parts. Prove the following.

(a) $\sum_{k=1}^m p_k(n) = p_m(n + m)$. [1]

(b) $p_k(n) = p_k(n - k) + p_{k-1}(n - 1)$. [2]

(c) $p_n(2n) = p(n)$. [2]

13. Using Theorem 13.2.2., calculate $p(10)$. [5]